[redacted]

[redacted]

[redacted]

DIFFERENTIAL REFRACTION IN POSITIONAL ASTRONOMY

DEPARTMENT OF MERIDIAN ASTROMETRY
CARNEGIE INSTITUTION OF WASHINGTON
Dudley Observatory, Albany, N. Y.
1922.

DIFFERENTIAL REFRACTION IN POSITIONAL ASTRONOMY

By WILLIAM B. VARNUM

Early in the nineteenth century meridian observers noticed certain systematic errors in star positions which were ascribed by Bessel and DeBall, among others, to variation in refraction, apparently of a seasonal nature. Little was done, however, to determine the nature and amount of these errors until the latter part of the century, when Lewis Boss, Auwers, and Newcomb, in forming their fundamental systems of positions, recognized the existence of systematic errors dependent upon right-ascension, the seasonal effect, and upon declination. Various suggestions were made as to the physical causes of these errors but, owing to the difficulty in defining the effects of the various sources of error on the observations, it became the common practice to eliminate them, as far as possible, by proper combinations of observations and by means of systematic corrections derived from comparison with a fundamental catalog upon the assumption that the systematic errors had been eliminated, or at least materially reduced, in the formation of the fundamental catalog by the combination of observations from various sources, taken with different instruments, under widely differing conditions and reduced by different methods. Thus were introduced into Positional Astronomy the well-known corrections $\Delta\alpha_\alpha$, $\Delta\alpha_\delta$, $\Delta\delta_\alpha$, and $\Delta\delta_\delta$. The two terms dependent upon α have the form ($a \sin \alpha + b \cos \alpha + c \sin 2\alpha + d \cos 2\alpha$).

In seeking an explanation of the Kimura term in the variation of latitude Ross (1) suggested in 1912 the possibility of a "secular refraction starting at sunset" or a seasonal refraction effect of the form

$$r = \alpha + \beta \cos \odot + \gamma \sin \odot$$

In the same year Tucker (2) published a discussion of the position of the mire at Mt. Hamilton deriving a set of empirical corrections necessary to reduce the observations of the early hours of the night to a standard value. In 1913 the same writer (3), in a paper on "Diurnal Variation of Refraction at Mt. Hamilton," established a difference in the effect of refraction between daytime and night observations, which might be expressed in terms of a correction of the Pulkova refraction constant. In his paper he says, "This difference does not depend upon barometric pressure, nor upon temperature nor upon the changes of temperature during the observing hours." Yet, later in the article, he offers a possible explanation of the phenomenon on the basis of the difference in the effects of temperature changes on the upper and lower air strata and notes that this supposition would bring daytime refractions into closer accord with the night.

In the same year the author found a similar diurnal term in the clock corrections derived from the 12-hour and 24-hour groups of stars at San Luis. In the Year Book of the Carnegie Institution of Washington for 1913 are given the values of this term for each hour of the day, for each of the four seasons of the year, and the mean values for the twenty-two months of observing at San Luis. In discussing these observations there did not seem to be sufficient justification for attempting to eliminate this term as the Riefler clock was not under control. When, however, the same term was found in clock corrections derived from the Albany observations where the new Reifler clock was running under perfect control, it appeared that the diurnal effect could not be due to the clock. This belief was strengthened by finding the same phenomenon in the Greenwich observations of 1907-8, where two clocks were employed; in the Pulkova observations of 1894-6, where, also, two clocks were used; and in the Cape 1900 observations. The results of these tests of observations made at other observatories were published in the Year Book of the Carnegie Institution for 1920. The same effect has been noted by Tucker at Mt. Hamilton and by Eichelberger at Washington where at least two clocks were used. Thus, in the observations of seven widely-separated institutions employing at least ten different clocks the

[1] *Astronomische Nachrichten 192*, 142, 1912.
[2] *Lick Observatory Bulletin* 7, 41, 1912.
[3] *Lick Observatory Bulletin* 7, 130, 1913.

same term has been found. It has been customary, heretofore, to ascribe the diurnal effect in declination to variation in refraction and that in transits to the clock. There appear to be no series of observations which are sufficiently inclusive to form the basis for a general discussion of the diurnal effect other than those of San Luis and Albany. In these two series, morning, afternoon, and night observations are made of all stars bright enough to be visible with sky illumination. Not being confined to a few selected stars, and those stars observed on selected dates, our observations are free from personal bias and give a true representation of the diurnal phenomenon. In the following tables are exhibited the form and values of this term for various combinations and stations.

In Table A are given the seasonal values of the term, together with the mean from all for the San Luis station. The first column gives the values derived from the curves while the second gives the computed values expanded by the formula given below. There can be no doubt that a sin MT + cos MT + sin 2 MT + cos 2 MT formula fits the curves very closely. In

TABLE A

DIURNAL TERM IN TRANSITS

San Luis Reifler No. 88

M. T.	Mar. – May Curve	Mar. – May Comp.	June – Aug. Curve	June – Aug. Comp.	Sept. – Nov. Curve	Sept. – Nov. Comp.	Dec. – Feb. Curve	Dec. – Feb. Comp.	Mean Curve	Mean Comp.
h	s	s	s	s	s	s	s	s	s	s
0	+0.004	+0.008	+0.018	+0.019	+0.012	+0.010	+0.028	+0.033	+0.016	+0.018
1	+ 6	+ 10	+ 25	+ 25	+ 13	+ 14	+ 31	+ 37	+ 19	+ 21
2	+ 8	+ 11	+ 30	+ 28	+ 14	+ 17	+ 34	+ 40	+ 22	+ 24
3	+ 10	+ 10	+ 32	+ 28	+ 16	+ 19	+ 39	+ 36	+ 24	+ 24
4	+ 14	+ 10	+ 27	+ 25	+ 18	+ 19	+ 44	+ 32	+ 26	+ 22
5	+ 14	+ 9	+ 13	+ 20	+ 19	+ 17	+ 37	+ 26	+ 21	+ 18
6	+ 10	+ 7	+ 6	+ 14	+ 19	+ 14	+ 19	+ 21	+ 14	+ 14
7	+ 4	+ 6	+ 13	+ 8	+ 12	+ 10	+ 5	+ 15	+ 8	+ 9
8	+ 4	+ 6	+ 8	+ 2	+ 2	+ 5	+ 4	+ 9	+ 4	+ 5
9	0	+ 5	− 2	− 2	− 10	0	+ 7	+ 4	− 1	+ 2
10	− 3	+ 4	− 6	− 4	− 7	− 4	+ 3	+ 1	− 3	− 1
11	+ 5	+ 3	− 6	− 6	− 3	− 7	− 4	− 7	− 2	− 4
12	+ 8	+ 1	− 7	− 7	− 4	− 10	− 11	− 12	− 4	− 6
13	+ 5	− 1	− 8	− 8	− 6	− 11	− 16	− 17	− 6	− 8
14	− 1	− 4	− 10	− 9	− 9	− 11	− 20	− 25	− 10	− 10
15	− 8	− 6	− 11	− 11	− 12	− 11	− 23	− 25	− 12	− 13
16	− 14	− 8	− 12	− 13	− 14	− 10	− 26	− 26	− 16	− 14
17	− 15	− 9	− 14	− 15	− 13	− 10	− 26	− 25	− 17	− 14
18	− 12	− 9	− 14	− 15	− 12	− 8	− 26	− 21	− 16	− 13
19	− 5	− 7	− 14	− 14	− 8	− 7	− 17	− 13	− 11	− 11
20	0	− 5	− 11	− 10	− 4	− 5	+ 4	− 4	− 3	− 6
21	+ 2	− 2	− 5	− 4	0	− 2	+ 16	+ 7	+ 3	0
22	+ 3	+ 1	+ 2	+ 3	+ 4	+ 2	+ 22	+ 9	+ 8	+ 6
23	+0.003	+0.003	+0.010	+ 0.011	+0.008	+0.007	+0.025	+0.017	+0.012	+0.013

		sin MT	cos MT	sin 2 MT	cos 2 MT
	s	s	s	s	s
Apr.	C = +0.0019	+0.0083	+0.0033	+0.0003	+0.0028
Jul.	= +0.0027	+0.0146	+ .0127	+ .0059	+ .0034
Oct.	= +0.0015	+0.0112	+ .0098	+ .0023	− .0013
Jan.	= +0.0052	+0.0206	+ .0226	+ .0000	+ .0052
All	= +0.0032	+0.0136	+ .0121	+ .0024	+ .0030

Table B are given the mean values for Albany and San Luis for the two Riefler clocks covering the whole period of observation. No. 88 was mounted three times and was never under control: No. 218 was mounted once and was always under perfect control: yet, these four conditions of the clock give similar forms of the diurnal effect. These values, also, correspond well to a single and double sine and cosine formula.

TABLE B

DIURNAL TERM IN TRANSITS

Albany Reiflers No. 88 and No. 218

M.T.	No. 88 1907 — 8 Obs'd	Comp.	1911 — 13 Obs'd	Comp.	No. 218 1915 — 18 Obs'd	Comp.	Mean S. Luis-Albany Obs'd	Comp.
h	s	s	s	s	s	s	s	s
0.5	....	....	....	....	+0.002	+0.004	+0.002	+0.004
1.5	+0.008	−0.005	+0.080	+0.050	+ 7	+ 6	+ 11	+ 7
2.5	− 19	0	....	....	− 5	+ 10	− 18	+ 10
3.5	− 3	+ 8	+ 47	+ 49	+ 4	+ 12	+ 5	+ 12
4.5	+ 27	+ 18	+ 40	+ 40	+ 22	+ 13	+ 24	+ 14
5.5	+ 47	+ 26	+ 19	+ 32	+ 21	+ 13	+ 24	+ 14
6.5	+ 63	+ 33	+ 21	+ 17	+ 13	+ 13	+ 20	+ 15
7.5	− 5	+ 35	+ 8	+ 12	+ 14	+ 13	+ 10	+ 15
8.5	+ 19	+ 31	+ 10	+ 11	+ 7	+ 13	+ 9	+ 14
9.5	+ 15	+ 20	+ 19	+ 13	+ 13	+ 13	+ 14	+ 14
10.5	+ 2	+ 6	+ 26	+ 20	+ 11	+ 13	+ 10	+ 13
11.5	+ 8	− 10	+ 27	+ 27	+ 10	+ 13	+ 12	+ 11
12.5	+ 1	− 23	+ 24	+ 31	+ 17	+ 12	+ 12	+ 9
13.5	− 23	− 34	....	....	+ 6	+ 10	+ 3	+ 6
14.5	− 70	− 41	....	....	+ 15	+ 7	+ 10	+ 2
15.5	− 57	− 41	....	....	+ 2	+ 3	− 7	− 2
16.5	− 32	− 37	....	....	− 3	− 3	− 7	− 7
17.5	− 31	− 28	....	....	− 18	− 8	− 20	− 10
18.5	− 7	− 21	− 15	− 12	− 5	− 11	− 6	− 12
19.5	− 7	− 13	− 22	− 15	− 12	− 14	− 12	− 13
20.5	− 10	− 9	+ 4	− 12	− 12	− 14	− 10	− 12
21.5	− 6	− 7	+ 11	− 4	− 18	− 12	− 13	− 10
22.5	−0.015	−0.004	+ 10	+ 11	− 19	− 8	− 15	− 6
23.5	....	....	−0.018	+0.022	+0.022	−0.003	+0.016	−0.002

	C	sin MT	cos MT	sin 2 MT	cos 2 MT
	s	s	s	s	s
1907-08,	= −0.0049	+0.0277	+0.0043	−0.0144	−0.0079
1911-13	= + .0201	+ .0158	+ .0015	+ .0187	+ .0110
1915-18	= + .0038	+ .0113	− .0067	+ .0039	+ .0021
1907-18	= + .0034	+ .0129	− .0046	+ .0018	+ .0018

In Table C are given the mean values of the four meteorological co-efficients of the atmosphere. These values are mean annual values for each hour of the day and are compiled from the reports of the New York Meteorological Observatory. The effect, also, of barometer and thermometer on the Pulkova refractions is given. The first column in each group gives the observed value, the second the diurnal term in the observed, and the third the diurnal term computed by the same formulæ used to represent the similar term in the clock-corrections. The diurnal term in zenith-distance, shown in a later table, is of similar form. In other words, the phenomenon found in the observed transits and the observed zenith-distances is of the same form as that shown to exist in the meteorology, a natural phenomenon of the form a sin MT + b cos MT + c sin 2MT + d cos 2MT. Therefore it would seem unreasonable to attribute systematic error in transits to as perfect a piece of mechanism as the modern astronomical clock and to seek a widely different reason for a similar term in zenith-distances. In view of the similarity between the diurnal term in transits and zenith-distances and the diurnal changes in the state of our atmosphere, is it not the most natural course to examine our observations for a refractional effect?

But to find a logical explanation of the diurnal term it is necessary to disregard the conclusions of many writers on refraction, to modify the theory on which all our refraction tables have been based and assume *that there is a varying prismatic effect due to the changes in the strata of our atmosphere.* This may appear rank heresy until the reader has digested the results presented in this paper but, once digested and absorbed, it must appear a very natural and logical explanation for part, at least, of this troublesome phenomenon. It is evidently wrong to assume that the strata of the atmosphere are horizontal with the *Earth's* surface and to neglect the consideration of a changing prismatic effect. In fact, examination of twenty-two series of observations extending over two years leads to the conclusion that the term "Anomalous Refraction" should really be applied to the rare case when there is no change in the prismatic effect of the atmosphere, that this prismatic effect gives rise to a differential of the vertical refraction and affects both our right-ascensions and declinations.

Let PV represent the prime vertical, MN, the meridian, and Z, the zenith. Imagine the atmosphere to be so constituted that, due to causes to be considered later, it produces a changing prismatic effect upon the rays of light from the star and let AB represent the direction of this prismatic displacement. Call μ the index of refraction, as it were, of the air. If μ remained constant we would have just $F\mu$ as a constant correction to the vertical refractions already applied. But we assume that μ varies with the time of day and we call the rate of change of μ, ρ. So we have a second term $F\mu\rho$. As will be seen from the figure, when ρ is positive the star is apparently at Z', while when ρ is minus the star is apparently at Z''. Or, for $+\rho$ the apparent meridian is east of the true MN, the stars transit too early and we have to apply

TABLE C

DIURNAL TERMS IN METEOROLOGY

M.T.	Barometer B	Barometer Diur.	Barometer Comp.	Shade Temp. T	Shade Temp. Diur.	Shade Temp. Comp.	Hygrometer H	Hygrometer Diur.	Hygrometer Comp.
h				°	°	°			
0	29.937	+0.007	+0.007	+56.2	+2.2	+2.4	68.0	−4.8	−4.9
1	.922	− 8	− 4	+57.2	+3.2	+3.5	66.0	−6.8	−7.1
2	.913	− 17	− 15	+58.4	+4.2	+4.3	64.5	−8.3	−8.6
3	.907	− 23	− 22	+58.6	+4.6	+4.6	63.7	−9.1	−9.2
4	.905	− 25	− 26	+58.6	+4.6	+4.5	63.6	−9.2	−8.9
5	.908	− 22	− 24	+58.1	+4.1	+4.0	64.4	−8.4	−7.8
6	.913	− 17	− 18	+57.2	+3.2	+3.1	66.3	−6.5	−6.1
7	.920	− 10	− 10	+56.0	+2.0	+2.2	69.0	−3.8	−4.0
8	.928	− 2	− 1	+55.0	+1.0	+1.2	71.4	−1.4	−1.9
9	.936	+ 6	+ 6	+54.2	+0.2	+0.3	73.2	+0.4	+0.1
10	.936	+ 6	+ 9	+53.6	−0.4	−0.7	74.5	+1.7	+1.8
11	.938	+ 8	+ 8	+53.0	−1.0	−1.3	75.7	+2.9	+3.2
12	.935	+ 5	+ 5	+52.4	−1.6	−1.7	76.9	+4.1	+4.3
13	.930	0	0	+51.9	−2.1	−2.3	77.8	+5.0	+5.2
14	.928	− 2	− 5	+51.4	−2.6	−2.8	78.9	+6.1	+6.0
15	.925	− 5	− 7	+51.0	−3.0	−3.2	79.3	+6.5	+6.6
16	.924	− 6	− 6	+50.6	−3.4	−3.5	79.9	+7.1	+7.1
17	.926	− 4	− 1	+50.4	−3.6	−3.7	80.4	+7.6	+7.3
18	.936	+ 6	+ 7	+50.3	−3.7	−3.7	80.2	+7.4	+7.1
19	.946	+ 16	+ 15	+50.8	−3.2	−3.4	79.2	+6.4	+6.3
20	.951	+ 21	+ 22	+51.7	−2.3	−2.7	77.4	+4.6	+4.9
21	.955	+ 25	+ 25	+52.8	−1.3	−1.7	75.2	+2.4	+2.9
22	.958	+ 28	+ 24	+53.8	−0.2	−0.5	73.5	+0.7	+0.4
23	.950	+0.020	+0.018	+55.0	+1.0	+1.1	70.3	−2.5	−2.3

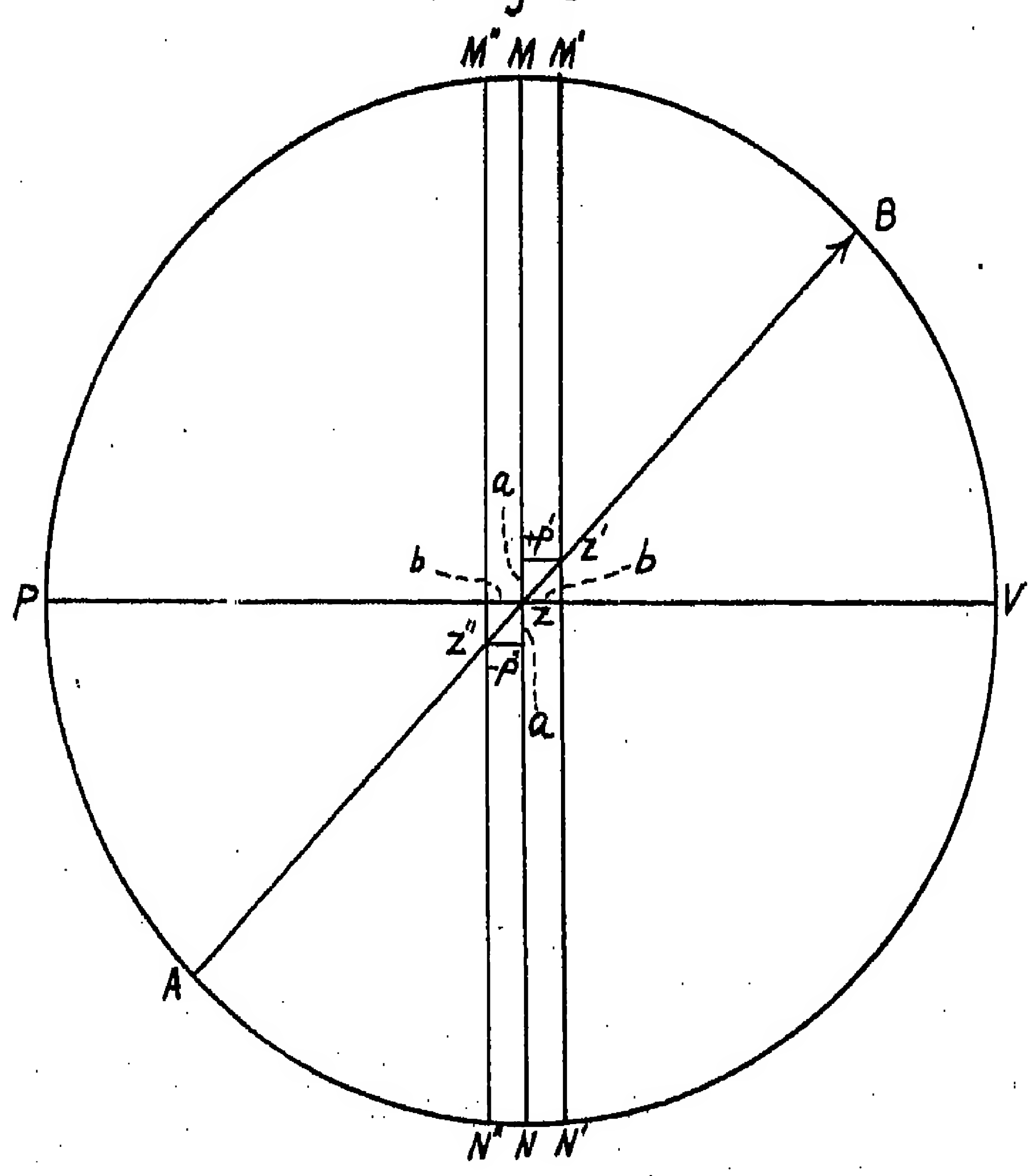
TABLE C

DIURNAL TERMS IN METEOROLOGY

M.T.	Sun Temp. S. T.	Sun Temp. Diur.	Sun Temp. Comp.	Shade Temp. Relative Refr. Refr.	Shade Temp. Relative Refr. ρ	Shade Temp. Relative Refr. Comp.	Sun Temp. Relative Refr. Refr.	Sun Temp. Relative Refr. ρ	Sun Temp. Relative Refr. Comp.
h	°	°	°						
0	+89.8	+24.5	+25.8	0.9971	−0.24	−0.23	0.9356	−0.26	−0.15
1	+91.0	+25.7	+26.3	47	− .27	− .15	.9330	+ .25	+ .43
2	+89.4	+24.1	+23.8	20	− .05	− .06	.9355	+ .86	+ .91
3	+84.4	+19.1	+18.6	15	− .01	+ .03	.9441	+1.20	+1.23
4	+77.6	+12.3	+11.7	14	+ .14	+ .12	.9561	+1.50	+1.32
5	+69.4	+ 4.1	+ 4.5	28	+ .16	+ .17	.9711	+1.35	+1.22
6	+62.3	− 3.0	− 2.1	44	+ .25	+ .20	.9846	+ .92	+ .96
7	+57.9	− 7.4	− 7.2	69	+ .22	+ .20	.9938	+ .53	+ .63
8	+55.0	−10.3	−10.5	0.9991	+ .19	+ .18	0.9991	+ .19	+ .32
9	+54.2	−11.1	−12.0	1.0010	+ .12	+ .15	1.0010	+ .12	+ .10
10	+53.6	−11.7	−12.3	22	+ .12	+ .12	.0022	+ .12	.00
11	+53.0	−12.3	−12.2	34	+ .11	+ .10	.0034	+ .11	+ .02
12	+52.4	−12.9	−12.2	45	+ .08	+ .09	.0045	+ .08	+ .10
13	+51.9	−13.4	−12.2	53	+ .09	+ .09	.0053	+ .09	+ .18
14	+51.4	−13.9	−13.6	62	+ .07	+ .08	.0062	+ .07	+ .19
15	+51.0	−14.3	−14.7	69	+ .08	+ .07	.0069	+ .08	+ .07
16	+50.6	−14.7	−15.1	77	+ .04	+ .04	.0077	− .04	− .18
17	+52.0	−13.3	−14.2	81	+ .06	.00	.0073	− .49	− .54
18	+53.5	−11.8	−11.3	87	− .07	− .06	1.0024	− .75	− .93
19	+57.5	− 7.8	− 6.3	80	− .16	− .14	0.9949	−1.34	−1.26
20	+64.6	− 0.7	+ .05	64	− .18	− .20	.9815	−1.71	−1.44
21	+74.0	+ 8.7	+ 8.3	46	− .45	− .26	.9644	−1.66	−1.41
22	+83.2	+17.9	+15.9	25	− .26	− .28	.9478	−1.05	−1.16
23	+87.8	+22.5	+22.1	1.0001	− .34	− .27	0.9373	− .17	− .71

			sin MT	cos MT	sin 2 MT	cos 2 MT
Barometer	Diurnal =	+0.0003	−0.0126	+0.0014	−0.0152	+0.0059
Shade Temp.	Diurnal =	+0.10	+3.33	+ 2.05	+0.71	+0.28
Hygrometer	Diurnal =	+0.09	−6.61	− 4.59	+1.40	−0.40
Sun Temp.	Diurnal =	+0.05	+4.58	+18.95	+1.89	+6.75
Shade Temp. ρ	Diurnal =	0.000	+0.133	− 0.159	+0.052	−0.068
Sun Temp. ρ	Diurnal =	−0.004	+0.943	− 0.127	+0.653	−0.018

Fig. I



a greater Δt to the transits, while for $-\rho$ the apparent meridan is west of MN, the stars transit too late and we have to apply a smaller Δt to the transits. Similar reasoning will apply to zenith-distances. The distances zz' and zz'' can be resolved into their rectangular coördinates a and b: a is considered the sine component and b the cosine component. The sine component is the shift in zenith-distance and the cosine component, the shift in right-ascension. *The total effect is essentially a shift of the meteorological zenith.*

Now let us consider what will be the effect of this assumption on the observed positions of the stars. Inasmuch as we have already dealt with vertical refraction in the zenith-distances, we will first take up the effect in that coördinate. The "constant" of refraction is not the same for two stations. It is cus-

tomary to apply to our observations a correction of the form

$$\Delta R = \text{True } R - \text{Computed } R_c = CR_c$$

where C is a constant and R_c is the computed refraction. To this we propose to add the differential of the vertical refraction, $dR = x \sec^2 z$, when the second order term is not taken into account. From the results of a preliminary investigation, however, it was found that the second order term became appreciable at low zenith-distance, hence

$$dR_1 = x \sec^2 z \; (1.00232 \; -.003486 \sec^2 z)$$

or

$$dR_1 = xF' \text{ where } F' = \sec^2 z \; (1.00232 - .003486 \sec^2 z)$$

F' can be tabulated once for all.

The next step is to find some connection between this formula and the state of the atmosphere. Those who have attempted to use a barometric gradient scaled off from a weather map have met with failure as was to be expected, for, as any practical astronomer knows, the rate of change of the barometer has little effect upon the computed refractions. It is the temperature change which is the controlling factor as far as a gradient is concerned. Compare refractions for some barometric gradient on a cold winter night with a hot summer night. The winter night gives an ascending gradient with high refractive power while the summer night gives an ascending gradient with low refractive power, which in itself would produce in our observations a seasonal effect. The *temperature*, then, and not the barometer, *is the controlling factor*. However, there is a much better way of using meteorology in connection with refraction. We have already tabulated the corrections to the log μ tan z of Pulkova. We have used them in computing vertical refractions, so they will surely be good enough to compute differential refraction, and the transformation is quite simple.

Let us assume that the Pulkova Tables represent 1.0 times the refraction at standard β and γ. Then applying log corrections, derived from Tables III, V, VII of Pulkova Tables, to the log 1.0 and taking out the natural number corresponding to the resulting logarithm, we get the relative refraction for our station for each period of observation, the value μ. From these relative values of the refraction we can form the hourly differences, or hourly rate of change of the refraction, ρ. In order to refer the ρ's on different nights to the same standard, we form $\mu\rho$. Both μ and $\mu\rho$ will produce a differential effect upon the zenith-distances, so we have as the full formula for investigating the observations

$$dR_1 = e'F'\mu + f'F'\mu\rho$$

The differential refraction will effect the observations with different signs depending on whether north or south zenith-distances are read: that is

for positions AE and BW, $dR_1 = +e'F'\mu + f'F'\mu\rho$
and for positions AW and BE, $dR_1 = -e'F'\mu - f'F'\mu\rho$
the vertical refractions, however, have had their signs changed to conform with tan z, so, if we use the formula with the positive sign, we will come out with $-dR$ for north zenith-distances, and, if we bear this in mind when we come to examine the effect of dR in the mean, we will not be led into error. We thus have for our complete correction, due to the atmosphere, in zenith-distance

$$dR = CR_c + e'F'\mu + f'F'\mu\rho \qquad (1)$$

In the formation of the normal equations $R_c/100$ and $100\ \rho$ were found to be more convenient than R and ρ, and were used.

Using the refractional value of the atmosphere as given for the zenith-distances, the form of the correction for differential refraction in right-ascension is

$$dR = e \cdot \sec z \cdot \sec \delta \cdot \mu + f \cdot \sec z \cdot \sec \delta \cdot \mu\rho$$

or, if we wish to take into account the second order terms,

$$dR = \sec \delta \cdot \sec z(1 - .00116 \sec^2 z) \; (e\mu + f\mu\rho)$$

or, letting

$$F = \sec \delta \cdot \sec z \; (1 -.00116 \sec^2 z)$$
$$dR = eF\mu + fF\mu\rho \qquad (2)$$

which is quite similar to the formula for zenith-distances. Formulæ (1) and (2), then, are the expressions which should be used to correct our observations for the prismatic effect of the *Earth's* atmosphere. The $F\mu$ term is the "drift" for the period under discussion and should account for a large part, if not all, of the Kimura, or z, term in the variation of latitude. It also takes out part of the "seasonal" effect. The $F\mu\rho$ term is the "diurnal" term and also takes out the rest of the "seasonal" effect.

Having thus derived the formulæ for the effect dR will have upon the observations in zenith-distance and right-ascension and having shown how to connect the formulæ with the meteorology, we next apply (1) and (2) to the twenty-two observational stretches selected for this tentative investigation, and derive the results shown in Table D. In this table are exhibited the Albany Mean Time; number of observations, original diurnal term, original diurnal term corrected for the various corrections treated of in this paper,

TABLE D

Albany Riefler No. 218

		Zenith Distance			Transits		
A. M. T.	Obs.	Orig. Diur.	Corrd. Diur.	C–O	Orig. Diur.	Corrd. Diur.	C–O
h m		″	″	″	″	″	″
23 44 D	3	+0.09	+0.35	+0.26	+0.41	+0.25	−0.16
1 16 D	6	−0.54	−0.22	−0.32	+0.15	+0.02	−0.13
2 8 D	10	+0.08	+0.07	−0.01	+0.32	+0.03	−0.29
3 3 D	26	+0.31	+0.06	−0.25	−0.11	−0.21	+0.10
4 3 D	93	−0.08	−0.11	+0.03	+0.26	+0.18	−0.08
5 0 D	55	+0.01	+0.06	+0.05	+0.38	+0.07	−0.31
5 56 D	30	−0.13	−0.25	+0.12	+0.35	+0.09	−0.26
7 6 D	43	+0.65	+0.09	−0.56	+0.47	+0.05	−0.42
8 4 D	22	+0.65	+0.10	−0.55	+0.71	+0.17	−0.54
8 43 D	1	+1.89	−0.56	−1.33	−0.03	−0.03	0.00
5 18 N	7	+0.18	+0.31	+0.13	+0.11	+0.41	+0.30
6 2 N	35	+0.39	+0.33	−0.06	−0.26	−0.08	−0.18
7 6 N	117	+0.46	+0.23	−0.23	−0.03	−0.06	+0.03
8 2 N	212	+0.50	+0.19	−0.31	−0.57	+0.02	−0.55
8 58 N	245	+0.35	+0.14	−0.21	+0.08	+0.01	−0.07
10 1 N	237	+0.37	+0.16	−0.21	+0.07	+0.07	−0.00
11 0 N	200	+0.32	+0.04	−0.28	+0.13	+0.06	−0.07
11 59 N	120	+0.57	+0.27	−0.30	−0.05	+0.10	+0.05
12 57 N	78	+0.27	+0.16	−0.11	−0.09	−0.08	−0.01
14 1 N	31	+0.19	+0.01	−0.18	+0.27	+0.30	+0.03
15 0 N	37	−0.04	+0.12	+0.08	+0.09	+0.06	−0.03
15 57 N	22	−0.10	+0.03	−0.07	+0.11	−0.04	−0.07
16 46 N	13	+0.25	+0.32	+0.07	+0.12	+0.04	−0.08
18 11 N	23	+0.08	0.00	−0.08	+0.10	+0.05	−0.05
18 32 N	5	−0.28	−0.21	−0.07	+0.51	+0.51	0.00
18 18 D	4	−0.48	−0.44	−0.04	−0.37	−0.22	−0.15
19 2 D	44	+0.22	+0.17	−0.05	+0.05	+0.16	+0.11
20 2 D	46	+0.14	−0.09	−0.05	−0.55	−0.31	−0.24
20 56 D	48	+0.29	+0.33	+0.04	−0.01	+0.07	+0.06
22 1 D	33	−0.10	−0.18	+0.08	−0.10	−0.19	+0.09
22 49 D	19	+0.23	+0.07	−0.16	−0.22	−0.26	+0.04
D PM	289	+0.13	+0.03	−0.10	+0.33	+0.09	−0.24
N	1382	+0.37	+0.10	−0.27	+0.03	+0.04	+0.01
D AM	194	+0.15	+0.07	−0.08	−0.17	−0.08	−0.09
D All	483	+0.14	+0.04	−0.10	+0.13	+0.02	−0.11
N − D		+0.23	+0.06	−0.17	−0.10	+0.02	−0.08

and (C — O). This last is in the sense of reducing the residuals numerically, and not algebraically, which accounts for the preponderance of minus signs. The residuals for transits are in the form $15''n \cos \delta$.

In Table E, the same residuals are used but no distinction is made as to day or night observations. It gives the original diurnal term, diurnal term computed from original by single and double sine and cosine formula, $(O\text{–}C)_1$, diurnal term corrected for dR, diurnal term computed from the corrected diurnal term as above, and $(O\text{–}C)_2$. In Table D we see that the three groups have been brought nearer together and the *(N–D) has been substantially reduced.*

For zenith distance in E we have

		sin MT	cos MT	sin 2 MT	cos 2 MT	
$\text{Comp}_1 =$	+0″.163	+0″.073	−0″.168	−0″.143	−0″.004	(Original)
$\text{Comp}_3 =$	+0 .082	+0 .087	−0 .109	−0 .110	−0 .020	$(+dR)$
$\text{Comp}_2 =$	+0 .082	−0 .014	−0 .060	−0 .034	+0 .016	(Residual)

For transits in E we have

$\text{Comp}_1 =$	+0 .057	+0 .074	+0 .003	+0 .130	+0 .007	(Original)
$\text{Comp}_3 =$	+0 .039	+0 .058	+0 .057	+0 .084	+0 .007	$(+dR)$
$\text{Comp}_2 =$	+0 .019	+0 .016	−0 .053	+0 .046	+0 .000	(Residual)

Comp._3 is derived from Table E_2 from columns Δn_δ and Δn_α, similarly to Comp._1 and Comp._2, by single plus double sine and cosine terms, to show that the dR term applied follows the general formula used in discussing the meteorological coefficients. $\text{Comp.}_1 - \text{Comp.}_3$ gives Comp._2 very closely.

For the investigation of the residual diurnal effect in R. A. and Decl., as shown under corrected diurnal effect in Tables E_3, it must be kept in mind that the Pulkova Tables do not take into account the relative humidity of the atmosphere and also that shade temperatures were used while, as a matter of fact, the daytime observations were not made in the shade but in sunshine. In order to show what effect the use of these terms may have on the diurnal term let us refer to Table C. We have μ_o from shade temperature and μ' from *Sun* temperature, and we can form $\rho' - \rho_o$, equals $\Delta\rho_o$, by subtracting ρ_o of the shade temperature from ρ' of *Sun* temperature. We can also form a ρ'' from the diurnal range of the hygrometer. We have used μ_o in our work on the Albany observations but we wish to use μ'. Let us form μ', $\mu'\Delta\rho_o$ and $\mu'\rho''$ as indicated above and solve, using the corrected, or residual, diurnal term of n_α and n_δ as given in Table E_3 for the n's. The form of the equation will then be

$$\mu' + \mu'\Delta\rho_o + \mu'\rho'' = n'_\alpha = n'_\delta$$

Solving,

$$\text{Corrd. Diur.} = n'_\delta = +0.044\,\mu' \quad -0.021\,\mu'\Delta\rho' \quad +0.013\,\mu'\rho''$$
$$\text{Corrd. Diur.} = n'_\alpha = +0.014 \quad +0.025 \quad +0.014$$

Substituting these values in the values of residual diurnal terms columns $(O\text{–}C)_4$ were obtained. Comparing $(O\text{–}C)_4$ of Table E_3 with $(O\text{–}C)_2$ of Table E_1 we find that out of 48 residuals, 45 agree in sign; from

TABLE E_1
ZENITH DISTANCES

A. M. T.		Orig. Diur.	$Comp._1$	$(O-C)_1$	Corrd Diur.	$Comp._2$	$(O-C)_2$
h	m	″	″	″	″	″	″
23	44	+0.09	+0.01	+0.08	+0.35	+0.04	+0.31
1	6	−0.54	−0.06	−0.48	−0.22	+0.02	−0.24
2	8	+0.08	−0.07	+0.15	+0.07	+0.00	+0.07
3	3	+0.31	−0.05	+0.36	+0.06	−0.00	+0.06
4	3	−0.08	+0.02	−0.10	−0.12	+0.00	−0.12
5	2	+0.03	+0.13	−0.10	+0.09	+0.02	+0.07
5	59	+0.15	+0.24	−0.09	−0.06	+0.05	−0.11
7	6	+0.51	+0.36	+0.15	+0.19	+0.09	+0.10
8	2	+0.51	+0.44	+0.07	+0.18	+0.12	+0.06
8	58	+0.36	+0.48	−0.12	+0.13	+0.15	−0.02
10	1	+0.37	+0.47	−0.10	+0.16	+0.16	0.00
11	0	+0.32	+0.41	−0.09	+0.04	+0.17	−0.13
11	59	+0.57	+0.33	+0.24	+0.27	+0.16	+0.11
12	57	+0.27	+0.24	+0.03	+0.16	+0.14	+0.02
14	1	+0.19	+0.15	+0.04	+0.01	+0.12	−0.11
15	0	−0.04	+0.09	−0.13	+0.12	+0.10	+0.02
15	57	−0.10	+0.06	−0.16	+0.03	+0.09	−0.06
16	46	+0.25	+0.06	+0.19	+0.32	+0.08	+0.24
18	12	0.00	+0.10	−0.10	−0.07	+0.05	−0.12
19	0	+0.16	+0.12	+0.04	+0.13	+0.08	+0.05
20	2	+0.14	+0.14	0.00	−0.09	+0.09	−0.18
20	56	+0.29	+0.14	+0.15	+0.33	+0.08	+0.25
22	1	−0.10	+0.10	+0.20	−0.18	+0.07	−0.25
22	49	+0.23	+0.06	−0.17	+0.07	+0.06	+0.01

TABLE E_2

A. M. T.		Zenith Distances $-\Delta n_\delta$	$Comp._3$	$(O-C)_3$	Transits $-\Delta n_\alpha$	$Comp._3$	$(O-C)_3$
h	m	″	″	″	″	″	″
23	44	−0.26	−0.04	−0.22	+0.16	+0.09	+0.07
1	6	−0.32	−0.08	−0.24	+0.13	+0.16	−0.03
2	8	+0.01	−0.07	+0.08	+0.29	+0.20	+0.09
3	3	+0.25	−0.04	+0.29	+0.10	+0.20	−0.10
4	3	+0.04	+0.02	+0.02	+0.08	+0.19	−0.11
5	2	−0.06	+0.10	−0.16	+0.24	+0.14	+0.10
5	59	+0.21	+0.19	+0.02	+0.02	+0.09	−0.07
7	6	+0.32	+0.27	+0.05	+0.13	+0.03	+0.10
8	2	+0.33	+0.32	+0.01	−0.03	−0.02	−0.01
8	58	+0.23	+0.33	−0.10	−0.09	−0.04	−0.05
10	1	+0.21	+0.30	−0.09	0.00	−0.05	+0.05
11	0	+0.28	+0.25	+0.03	+0.07	−0.04	+0.11
11	59	+0.31	+0.17	+0.13	−0.15	−0.01	−0.14
12	57	+0.11	+0.10	+0.01	−0.01	+0.02	−0.03
14	1	+0.18	+0.03	+0.15	−0.03	+0.04	−0.07
15	0	−0.16	−0.01	−0.15	+0.03	+0.04	−0.01
15	57	−0.13	−0.02	−0.11	+0.15	+0.03	+0.12
16	46	−0.07	−0.02	−0.05	+0.08	+0.01	+0.07
18	12	+0.07	+0.02	+0.05	+0.02	−0.03	+0.05
19	0	+0.03	+0.04	−0.01	−0.10	−0.05	−0.05
20	2	+0.23	+0.06	+0.17	−0.24	−0.06	−0.18
20	56	−0.04	+0.05	−0.09	−0.08	−0.05	−0.03
22	1	+0.08	+0.03	+0.05	+0.09	−0.01	+0.10
22	40	+0.16	−0.00	+0.16	+0.04	+0.03	+0.01

TABLE E_1
TRANSITS

A. M. T.		Orig. Diur.	$Comp._1$	$(O-C)_1$	Corrd Diur.	$Comp._2$	$(O-C)_2$
h	m	″	″	″	″	″	″
23	44	+0.41	+0.04	+0.37	+0.25	−0.04	+0.21
1	6	+0.15	+0.16	−0.01	+0.02	−0.00	+0.02
2	8	+0.32	+0.22	+0.10	+0.03	+0.02	+0.01
3	3	−0.11	+0.24	−0.35	−0.21	+0.04	−0.25
4	3	+0.27	+0.23	+0.04	+0.19	+0.05	+0.14
5	2	+0.35	+0.19	+0.16	+0.11	+0.04	+0.07
5	59	+0.02	+0.12	−0.10	0.00	+0.04	−0.04
7	6	+0.10	+0.05	+0.05	−0.03	+0.02	−0.05
8	2	+0.01	+0.00	+0.01	+0.04	+0.02	+0.02
8	58	−0.08	−0.02	−0.06	+0.01	+0.02	−0.01
10	1	+0.07	−0.02	+0.09	+0.07	+0.03	+0.04
11	0	+0.13	+0.01	+0.12	+0.06	+0.05	+0.01
11	59	−0.05	+0.06	−0.11	+0.10	+0.07	+0.03
12	57	−0.09	+0.10	−0.19	−0.08	+0.09	−0.17
14	1	+0.27	+0.13	+0.14	+0.30	+0.10	+0.20
15	0	+0.09	+0.13	−0.04	+0.06	+0.09	−0.03
15	57	+0.11	+0.10	+0.01	−0.04	+0.07	−0.11
16	46	+0.12	+0.06	+0.06	+0.04	+0.05	−0.01
18	12	+0.03	−0.04	+0.07	+0.01	−0.00	+0.01
19	0	+0.10	−0.08	+0.18	+0.20	−0.03	+0.23
20	2	−0.55	−0.12	−0.43	−0.31	−0.06	−0.25
20	56	−0.01	−0.12	+0.11	+0.07	−0.08	+0.15
22	1	−0.10	−0.08	−0.02	−0.19	−0.08	−0.11
22	49	−0.22	−0.03	−0.19	−0.26	−0.06	−0.20

TABLE E_3

M.T.	Zenith Distances Corr. Diur.	$Comp._4$	$(O-C)_4$	$(O-C)_2$	Transits Corr. Diur.	$Comp._4$	$(O-C)_4$	$(O-C)_2$
h	″	″	″	″	″	″	″	″
23.7	+0.35	−0.04	+0.39	+0.31	+0.25	+0.01	+0.24	+0.21
1.1	− .22	.00	− .22	− .24	+ .02	+ .01	+ .01	+ .02
2.1	+ .07	+ .02	+ .05	+ .07	+ .03	.00	+ .03	+ .01
3.0	+ .06	+ .05	+ .01	+ .06	− .21	+ .01	− .22	− .25
4.0	− .12	+ .07	− .19	− .12	+ .19	+ .02	+ .17	+ .14
5.0	+ .09	+ .08	+ .01	+ .07	+ .11	+ .04	+ .07	+ .07
6.0	− .06	+ .09	− .15	− .11	.00	+ .06	− .06	− .04
7.1	+ .19	+ .11	+ .08	+ .10	− .03	+ .09	− .12	− .05
8.0	+ .18	+ .10	+ .08	+ .06	+ .04	+ .09	− .05	+ .02
9.0	+ .13	+ .08	+ .05	− .02	+ .01	+ .10	− .09	− .01
10.0	+ .16	+ .06	+ .10	.00	+ .07	+ .09	− .02	+ .04
11.0	+ .04	+ .06	− .02	− .13	+ .06	+ .09	− .03	+ .01
12.0	+ .27	+ .06	+ .21	+ .11	+ .10	+ .08	+ .02	+ .03
13.0	+ .16	+ .06	+ .10	+ .02	− .08	+ .08	− .16	− .17
14.0	+ .01	+ .06	− .05	− .11	+ .30	+ .08	+ .22	+ .20
15.0	+ .12	+ .05	+ .07	+ .02	+ .06	+ .08	− .02	− .03
16.0	+ .03	+ .03	.00	− .06	− .04	+ .08	− .12	− .11
16.8	+ .32	+ .02	+ .30	+ .24	+ .04	+ .09	− .05	− .01
18.2	− .07	− .01	− .06	− .12	+ .01	+ .08	− .07	+ .01
19.0	+ .13	− .04	+ .17	+ .05	+ .20	+ .08	+ .12	+ .23
20.0	− .09	− .07	− .02	− .18	− .31	+ .07	− .38	− .25
20.9	+ .33	− .08	+ .41	+ .25	+ .07	+ .06	+ .01	+ .15
22.0	− .18	− .06	− .12	− .25	− .19	+ .05	− .24	− .11
22.8	+ .07	− .05	+ .12	+ .01	− .26	+ .03	− .29	− .20

which we conclude that *if humidity and the Sun temperature had been employed in the original solutions for dR the diurnal term would have been completely eliminated.* And this relation between Comp.$_2$ of Table E_1 and Comp.$_4$ of Table E_3 leads to the most important fact of this entire preliminary investigation — the fact that *the diurnal term is due directly to the atmosphere and that its law is*

$$\text{Diurnal term} = +F\mu\,[a \sin(\alpha - \odot) + b\cos(\alpha - \odot) + c \sin 2(\alpha - \odot) + d\cos 2(\alpha - \odot)]$$

where $\odot$ = apparent R. A. of the *Sun*.

Having thus the law of the diurnal term we can compute the correction for the diurnal term from our observations without knowing the humidity, or the *Sun* temperature. In fact it appears by actual trial that the formula integrates the varying conditions of air in upper and lower levels, and is therefore preferable to the use of a temperature derived solely from the conditions at the *Earth's* surface. But to make it clear to all that we have established this relation, perhaps it will be well to recapitulate and present in a concise form the reasons for this important conclusion.

We will assume that the expressions for dR_1 are accepted. That is,

$$dR_1 = x \cdot \sec\delta \cdot \sec z\,(1 - .00116 \sec^2 z) \text{ in R. A.}$$
$$dR_1 = x \cdot \sec^2 z\,(1.00232 - .003486 \sec^2 z) \text{ in Decl.}$$

We have shown conclusively that the expression

$$a \sin \mathrm{MT} + b \cos \mathrm{MT} + c \sin 2\,\mathrm{MT} + d \cos \mathrm{MT}$$

represents

NATURAL PHENOMENA	OBSERVATIONS
Barometer	San Lius Diurnal
Shade Temperature	Albany Diurnal
Sun Temperature	Albany Residual Diurnal
Hygrometer	
Shade Refraction	
Sun Refraction	

Having shown, not only theoretically but practically, that all of these natural phenomena follow one and the self-same law and that when the formulæ for dR_1 are combined with the formulæ for meteorology and vertical refraction we eliminate the diurnal term in the observations, are we not forced to the conclusion that we have discovered the physical explanation of the diurnal term which has been so troublesome in positional astronomy? With the preceding brief demonstration of the physical relation between the diurnal term and the state of the atmosphere, and the important part that differential refraction has played in enabling us to free our observations of the diurnal effect, it may be of interest to show the effect of *dR* upon the various systematic errors generally accepted as being inherent in any series of observations.

The following stretches were chosen from a consideration of the effect of *dR* in R. A. alone. A fair distribution during the year with Clamp East and Clamp West was sought. Due to the fact that the resulting effect was unknown little effort was made to select for a perfect distribution between observers and positions of the instrument. When it was discovered that the *dR* was a real determinable quantity so far as the R. A.'s were concerned, we decided to examine the Z. D.'s for the same stretches. If the effect was really due to *dR*, the Z. D.'s would prove it, while, if it was not due to *dR*, the Z. D.'s would disprove it. The series selected were

Series	Pos.	Obs'r	Time	Date
666–68	*AE*	S.A.	42^h	Aug. 31 to Sept. 1, 1915
669–70	*AE*	S.A.	27	Sept. 2 to Sept. 3, 1915
684–85	*AE*	W.B.V.	30	Sept. 27 to Sept. 28, 1915
686–87	*AE*	W.B.V.	41	Sept. 29 to Sept. 30, 1915
691–94	*AE*	S.A.	67	Oct. 10 to Oct. 13, 1915
700–04	*AW*	Roy	96	Oct. 26 to Oct. 30, 1915
711–14	*AE*	Roy	62	Nov. 15, to Nov. 18, 1915
728–31	*AE*	S.A.	63	Jan. 5 to Jan. 8, 1916
735–37	*AW*	S.A.	33	Jan. 23 to Jan. 25, 1916
739–40	*AW*	W.B.V.	43	Feb. 3 to Feb. 4, 1916
754–57	*AW*	S.A.	86	Mar. 28 to Mar. 31, 1916
760–61	*AE*	Roy	29	Apr. 9 to Apr. 10, 1916
771–74	*AW*	S.A.	65	May 10 to May 13, 1916
784–87	*BE*	S.A.	66	June 26 to July 1, 1916
815–18	*BW*	W.B.V.	80	Aug. 28 to Aug. 31, 1916
825–28	*BW*	W.B.V.	65	Sept. 18 to Sept. 21, 1916
855–57	*BW*	S.A.	41	Dec. 6 to Dec. 8, 1916
866–68	*BE*	Roy	38	Jan. 22 to Jan. 24, 1917
883–85	*BE*	S.A.	51	Mar. 18 to Mar. 20, 1917
903–05	*BE*	S.A.	87	May 13 to May 17, 1917
948–49	*AE*	W.B.V.	40	Nov. 11 to Nov. 12, 1917
950–51	*AE*	W.B.V.	30	Nov. 13 to Nov. 14, 1917

For brevity, we will use the first series in each stretch to designate the stretch. Deriving μ and ρ from Albany meteorology, and solving each stretch according to equations (1) and (2), the following values of coefficients in the formulæ for *dR* were obtained. At the same time these equations were solved, a solution was made for the correction to refraction accord-

ing to former practice; $n = CR/100$. These values are given under column CR only. These values have been applied to the n's throughout the work on Z. D.'s, giving the opportunity to examine the true effect of using dR as contrasted with CR only, so that in Z. D. we always have three sets of n's; one original n, one $n + CR$, and one $n + dR$. In R. A. we have original n, and $n + dR$.

	R.A.			Z.D.		
Stretch	e	f	CR only	C	e'	f'
	s	s	″	″	″	″
666	+0.0105	+0.0120	+0.737	+0.484	+0.163	−0.004
669	− .0034	+ .0147	+ .301	+ .115	+ .118	− .081
684	+ .0008	+ .0081	− .111	− .071	− .040	+ .016
686	− .0014	− .0113	− .173	+ .014	− .080	+ .010
691	+ .0096	+ .0212	+ .202	− .341	+ .161	− .046
700	− .0092	− .0014	+ .096	+ .244	+ .057	+ .015
711	+ .0041	+ .0098	−0.026	+ .267	− .079	− .009
728	− .0019	+ .0159	+1.352	+ .972	+ .144	− .008
735	− .0032	+ .0101	+0.402	+ .443	− .002	+ .129
730	+ .0002	+ .0101	− .318	− .219	− .010	+ .192
754	+ .0038	+ .0014	− .244	− .140	− .028	+ .139
760	− .0039	+ .0033	−0.219	− .056	− .158	+ .088
771	+ .0004	+ .0068	−1.251	− .552	+ .217	+ .191
784	+ .0035	+ .0240	−0.469	− .484	− .047	+ .119
815	− .0045	− .0249	+ .404	+ .116	+ .147	+ .044
825	+ .0018	− .0004	− .079	− .058	− .069	+ .219
855	− .0014	− .0002	+ .331	+ .337	+ .026	− .191
866	− .0080	− .0063	+ .120	+ .156	+ .009	+ .110
883	+ .0027	+ .0178	− .437	− .336	+ .096	− .301
903	+ .0103	+ .0117	− .749	− .504	+ .155	− .272
948	+ .0089	− .0277	− .082	+ .011	− .089	+ .361
950	+ .0022	+ .0025	− .258	− .175	− .067	+ .024

Having expanded these values and corrected the original n's we obtained $n + dR = n'$ for Z. D.'s and Transits. Both the original n's and the n's corrected for dR contain the systematic errors $\Delta\alpha_\alpha$, $\Delta\alpha_\delta$, $\Delta\delta_\alpha$, $\Delta\delta_\delta$ and U–L, except in so far as the application of dR has eliminated them in the n's. Let us see what the effect of dR has been. Using zones 3° wide, means for $\Delta\delta_\delta$ and $\Delta\alpha_\delta$ were formed. The means of successive groups of three were used to smooth the curve, and these are exhibited in Plate A. The drawing contains sufficient explanation to enable one to understand why they differ but attention should be called to the effect of dR upon the curves, especially the $\Delta\delta_\delta$ curve. The circles represent the results from the n's in order of Z. D. from horizon to horizon. The crosses represent the values derived from below pole observations folded back and transformed to correspond to above pole observations. There can be but one true value of $\Delta\alpha_\delta$ and $\Delta\delta_\delta$ for a given star; any deviation must be due to the observations themselves. While the original observations contain some (U–L), when they are corrected for CR we find this (U–L) is very markedly increased, but when corrected for dR *the* (U–L) *has practically disappeared,* reconciling the observations below and above pole. This alone is an indication that dR is real. The wider deviation from zero of the curve at the south end may be due to systematic error in *P. G. C.* or to the false form of vertical refraction formulæ; it is probably due to a combination of both. This point, however, cannot be investigated until the San Luis observations for dR have been examined. Then we hope to be able to indicate the probable source of this wide divergence from zero. Having plotted these points, smooth curves were drawn, and a table formed. That for $\Delta\delta_\delta$ was applied to the Z. D.'s. That for $\Delta\alpha_\delta'$ was combined with (E–W). The intimate relation between (E–W) and $\Delta\alpha_\delta$ led to the use of $n' \cos\delta$ in place of n', so

$$[\text{Curve } \Delta\alpha_\delta + (E\text{–}W)] \sec\delta = \text{correction.}$$

Correcting Z. D.'s and Transits for these values, thus freeing the observations from the systematic errors $\Delta\alpha_\delta$ and $\Delta\delta_\delta$, we derived n'' from which to obtain $\Delta\alpha_\alpha$ and $\Delta\delta_\alpha$. Hourly means were formed and solved using the clock belt only. This gave

$$\begin{array}{llllll} & & \sin & \cos & \sin 2 & \cos 2 \\ & & \text{s} & \text{s} & \text{s} & \text{s} \\ n & = -\Delta\alpha_\alpha = & +0.0026 & -0.0194 & +0.0115 & -0.0014 \text{ Orig.} \\ n'' & = -\Delta\alpha_\alpha = & -0.0006 & -0.0080 & +0.0058 & -0.0045 \text{ Corrd.} \\ & & '' & '' & '' & '' \\ n & = -\Delta\delta_\alpha = & -0.104 & -0.070 & +0.117 & -0.049 \text{ Orig.} \\ n'' & = -\Delta\delta_\alpha = & -0.016 & -0.012 & +0.006 & -0.010 \text{ Corrd.} \end{array}$$

The values derived from n'' show the effect of application of dR to original n. *The systematic errors $\Delta\alpha_\alpha$ and $\Delta\delta_\alpha$ have been practically eliminated.* It must be noted here that we have used the value of dR based upon a value of ρ computed from imperfect meteorology. If we compute ρ by formula, as indicated earlier in this paper, we obtain complete elimination of the two systematic errors depending upon right-ascension. These values from computed n'' were expanded and applied to observed n'' giving n''' free from all systematic errors. The n's and n'''s were collected in order of Albany Mean Time giving the diurnal term in Table D treated earlier in this paper. Comparisons of the effect of dR with that of CR on the Z. D.'s are given in Tables F and G. These show that dR *brings the observations north and south of the zenith more into agreement, without the application of a constant.* Table F gives results of application of CR and of dR for each series. It is to be noted that the application of dR has reduced the p. e. by only 0″.01 in the mean; which shows that we are correcting for the systematic shift of the stars and not for the accidental error of the observations. Table G shows how erroneous it is to correct our observations for CR only. When CR only is applied, all the residuals north of the zenith are in-

TABLE F NORTH STARS

Obs.	n	$n+CR$	$n+dR$	(1)	(2)	(3)	(4)
	″	″	″	″	″	″	″
26	+ 7.71	+22.58	− 0.96	+11.51	−0.53	−23.54	−12.04
20	+ 8.15	+12.68	+ 1.78	+ 1.67	−1.25	−10.90	− 2.92
39	− 5.37	− 4.24	− 1.76	+ 1.19	+0.69	+ 2.48	− 0.50
37	− 9.84	− 9.84	+ 1.31	+ 1.38	+5.83	+11.15	+ 4.45
36	+18.29	+21.09	+ 2.95	+ 1.52	−3.04	−18.14	− 4.56
69	+11.87	+ 8.51	− 6.19	− 0.50	+0.50	−14.70	+ 1.30
69	−27.74	−28.46	−10.58	+ 0.22	−2.31	+17.88	− 2.53
32	+ 3.46	+25.05	+ 7.95	+11.29	+0.47	−17.10	−10.82
16	+ 8.37	+ 4.45	+ 3.84	− 1.60	−2.23	− 0.61	− 0.63
30	+ 1.00	+ 8.33	− 0.61	+ 1.43	−1.67	− 8.94	− 3.10
75	+12.25	+21.15	+ 4.13	+ 1.15	+3.18	−17.02	+ 2.03
33	−10.92	−15.03	− 0.96	+ 1.51	−0.26	+14.07	− 1.77
40	+16.88	+41.69	+ 2.03	+14.54	−1.31	−39.66	−15.85
47	− 4.23	+ 9.51	+10.26	+ 0.38	+0.98	+ 0.75	+ 0.60
67	+42.79	+56.72	+ 8.98	+13.05	−9.11	−47.74	−22.16
54	− 0.16	− 1.80	− 2.41	0.00	+5.63	− 0.61	+ 5.63
20	− 3.49	+ 0.56	+ 0.27	− 1.19	−1.60	− 0.29	− 0.41
22	+ 4.28	+ 2.49	+ 1.48	− 0.25	−1.18	− 1.01	− 0.93
43	+ 2.00	+13.77	+ 2.85	+ 2.17	−0.76	−10.92	− 2.93
22	+ 2.85	+11.20	+ 1.99	+ 3.01	−0.68	− 9.21	− 3.69
30	− 2.40	− 3.99	− 1.46	− 0.29	+1.26	+ 2.53	+ 1.55
35	+ 4.24	− 8.56	− 0.22	+ 0.46	+1.10	+ 8.34	+ 0.70
862	+71.51	+187.86	+24.67	+62.65	−5.93	−163.19	−68.58
Means	+ 0.08	+ 0.22	+ 0.03	+ 0.07	−0.01	− 0.19	− 0.08

TABLE F SOUTH STARS

Obs.	n	$n+CR$	$n+dR$	(1)	(2)	(3)	(4)
	″	″	″	″	″	″	″
58	+ 48.11	+21.95	+ 0.79	−10.58	− 4.00	−21.16	+6.58
40	+ 24.60	+17.06	+ 9.47	− 2.70	− 1.91	− 7.59	+0.79
83	+ 3.22	+ 1.64	+10.57	− 4.92	− 0.23	+ 8.93	+4.69
90	− 19.82	−20.69	+ 7.82	− 2.87	− 5.56	+28.49	−2.69
86	+ 35.95	+25.39	+14.87	− 2.21	+ 0.68	−10.52	+2.89
121	+ 10.15	+17.73	+ 7.45	+ 1.02	+ 0.90	−10.28	−0.12
131	− 18.22	−15.39	− 1.10	− 0.30	− 0.39	+14.29	−0.09
74	+ 99.76	+34.45	+17.72	−45.04	−38.68	−16.73	+6.36
51	− 4.35	+ 9.46	+ 9.56	+ 0.09	+ 0.15	+ 0.10	+0.06
74	+ 32.32	+18.50	+12.89	− 6.28	− 8.19	− 5.61	−1.91
108	+ 31.32	+13.66	− 1.19	− 3.18	+ 4.87	−14.85	+8.05
57	− 22.34	−13.08	+11.62	− 5.00	+ 0.94	+24.70	+5.94
121	+115.83	+36.33	− 1.13	−47.10	−40.94	−37.46	+6.16
155	+ 43.76	− 2.87	− 2.52	− 6.97	− 4.73	+ 0.35	+2.24
154	+ 99.39	+61.09	+18.53	−15.87	−12.44	−42.56	+3.43
127	− 9.74	− 2.33	− 3.41	− 0.47	− 7.03	− 1.08	−6.56
71	+ 20.22	+ 8.98	+ 8.18	− 0.72	− 1.62	− 0.80	−0.90
74	+ 4.16	+ 9.35	+ 7.32	− 0.54	+ 1.17	+ 2.03	+1.71
80	+ 41.85	+20.59	+10.88	− 2.24	+ 2.49	− 9.71	+4.73
46	+ 40.21	+10.03	+ 4.05	−17.46	−16.16	− 5.98	+1.30
73	+ 5.68	+ 9.15	+ 7.58	− 0.05	− 1.21	− 1.57	−1.15
66	− 20.47	−10.15	+ 0.78	− 4.92	− 5.53	+10.93	−0.61
1940	+561.6	+250.8	+150.7	−178.3	−137.4	−100.1	+40.9
Means	+ .29	+ .13	+ .08	− .09	− .07	− .05	+ .02

creased while all the residuals south of the zenith, except the 78° group, are decreased. This is shown in column (1). When, however, we correct n for dR we find the n's are decreased except for the low Z. D.'s each side of the zenith. The appearance of plus signs at each end of column (2) indicates that the formula for vertical refraction may be wrong. This is entirely consistent as all present tables are founded on observations uncorrected for dR. To be sure, when the observations of circumpolar stars have been empirically corrected for ($U–L$) they have, to a certain extent, been freed from the mean effect of dR. But the particular effect of dR for each stretch has not been taken into account as it should have been.

Table H, which is self-explanatory, shows the effect of dR upon the positions of some of the 19 Primary Azimuth Stars as deduced from the double transits on the stretches employed in this investigation.

Columns (2) and (3) give corrections to places of the 19 Primary Azimuth Stars derived from Albany observations. These corrections are derived from the double transits of the stretches used, corrected for dR. Columns (4) and (5) are the same corrections uncorrected for dR. Column (6) contains the value of $U–L$ derived from corrected values, while (7) is $U–L$ from uncorrected values. Column (8) shows the amount by which $U–L$ in (7) is reduced by dR as given in (6). This is computed in the sense of numerical reduction of $U–L$ and not algebraical reduction, so it indicates the reduction of the probable error of transits of the 19 Azimuth Stars by application of dR, and points to the principal source of ($U–L$).

The effect of dR upon equator point is equally pronounced. This can be well shown by a comparison of the mean equator point derived from the double tran-

TABLE H

No. P.G.C.	(2) U	(3) L	(4) U	(5) L	(6) U–L	(7) U–L	(8)
	s	s	s	s	s	s	s
325	+ .098	+ .050	+.115	+.175	+.030	−.107	−.597
1801	− .089	+ .051	+.025	+.026	−.003	+.150	−.231
1871	− .003	+ .009	+.028	+.032	−.013	−.004	−.071
2135	− .194	− .340	−.436	−.253	+.147	−.184	−.036
2536	− .051	− .024	−.050	+.012	−.002	−.043	−.021
4327	+ .008	+ .009	+.019	−.015	−.029	+.014	+.015
4591	+ .125	+ .093	+.151	+.038	+.026	+.060	−.112
4971	−1.065	−1.129	−.336	−.910	+.064	+.574	−.510
5499	− .158	− .035	−.178	+.242	−.122	−.420	−.298
218	+ .135	+ .135	+.270	+.364	.000	−.094	−.094

PLATE A

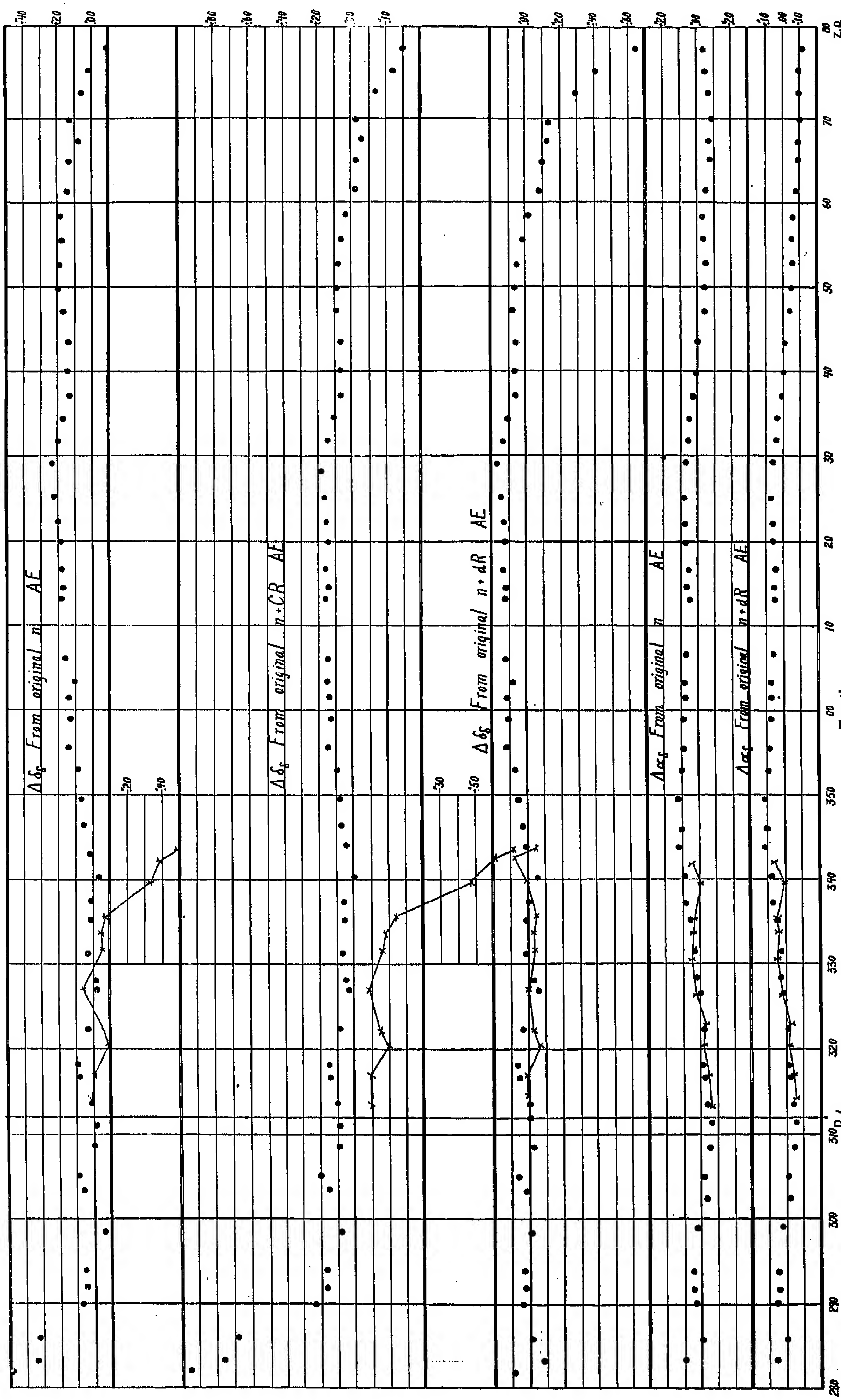
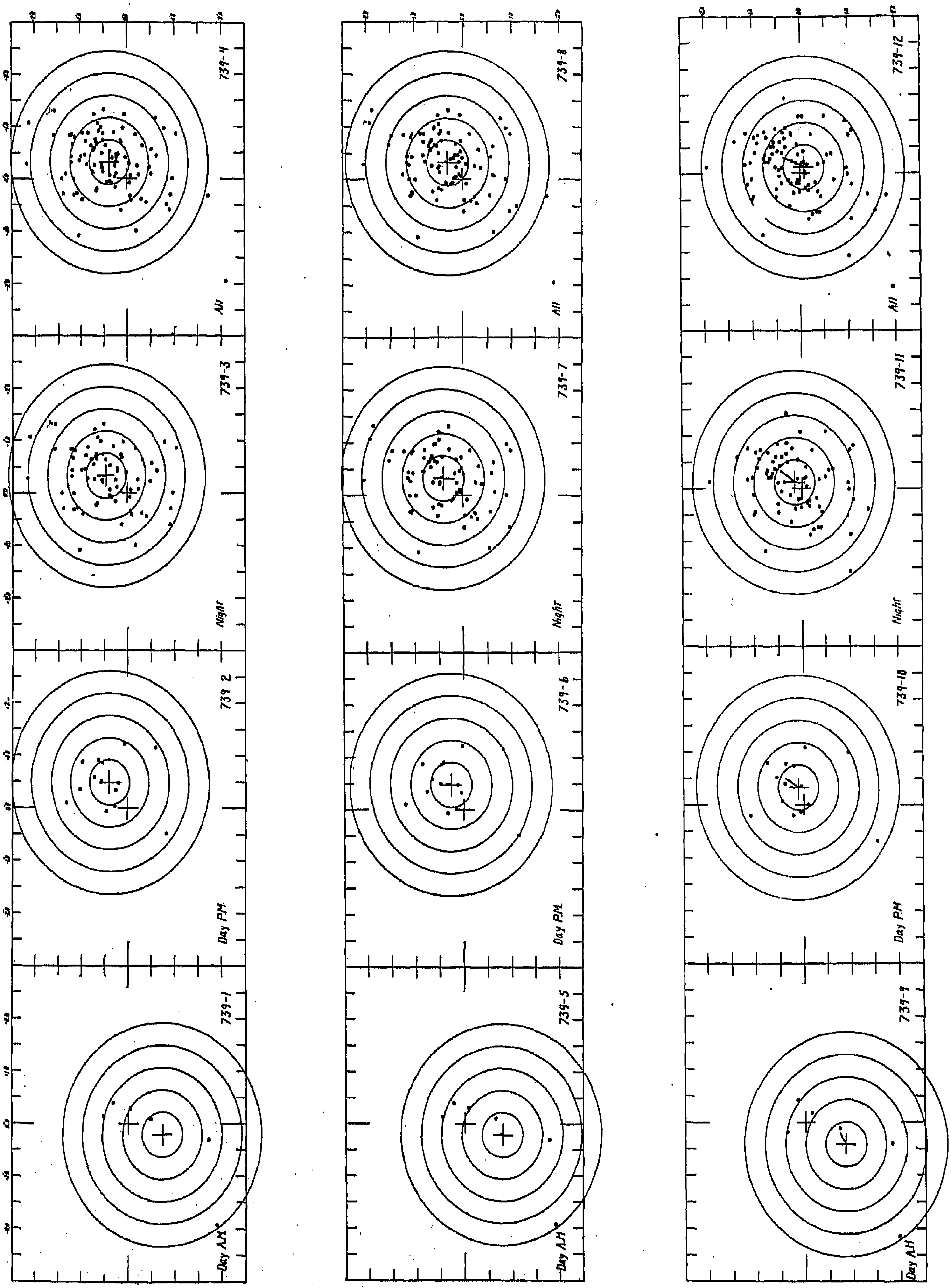

Δδs From original n AE
Δδs From original n+CR AE
Δδs From original n+dR AE
Δαs From original n AE
Δαs From original n+dR AE
Zenith
Pole
Z.D.

PLATE B



739-4
All
739-3
Night
739 2
Day P.M.
739-1
Day A.M.
739-8
All
739-7
Night
739-6
Day P.M.
739-5
Day A.M.
739-12
All
739-11
Night
739-10
Day P.M.
739-9
Day A.M.

PLATE C

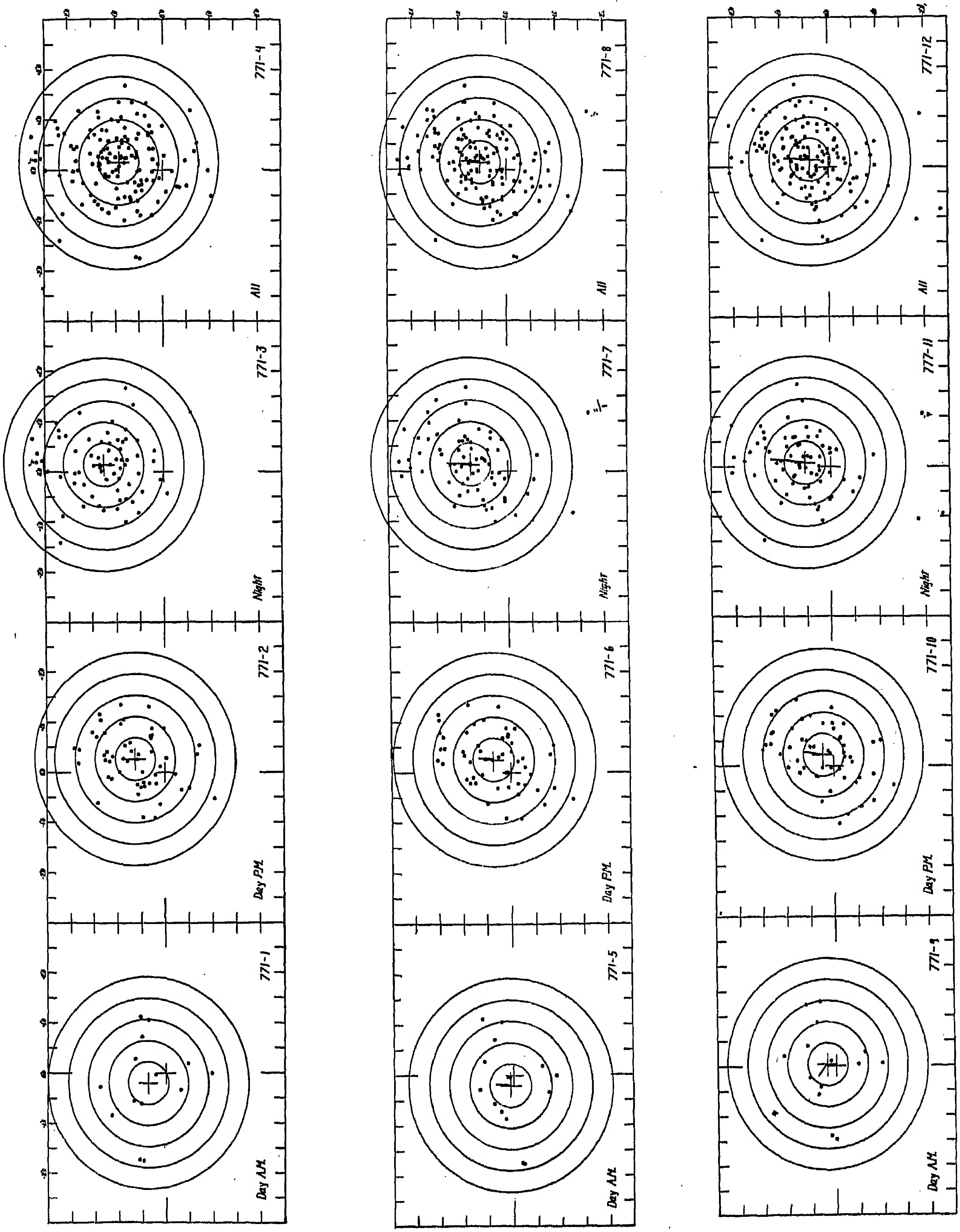

771-4
All
771-3
Night
771-2
Day P.M.
771-1
Day A.M.
771-8
All
771-7
Night
771-6
Day P.M.
771-5
Day A.M.
771-12
All
771-11
Night
771-10
Day P.M.
771-9
Day A.M.

PLATE D

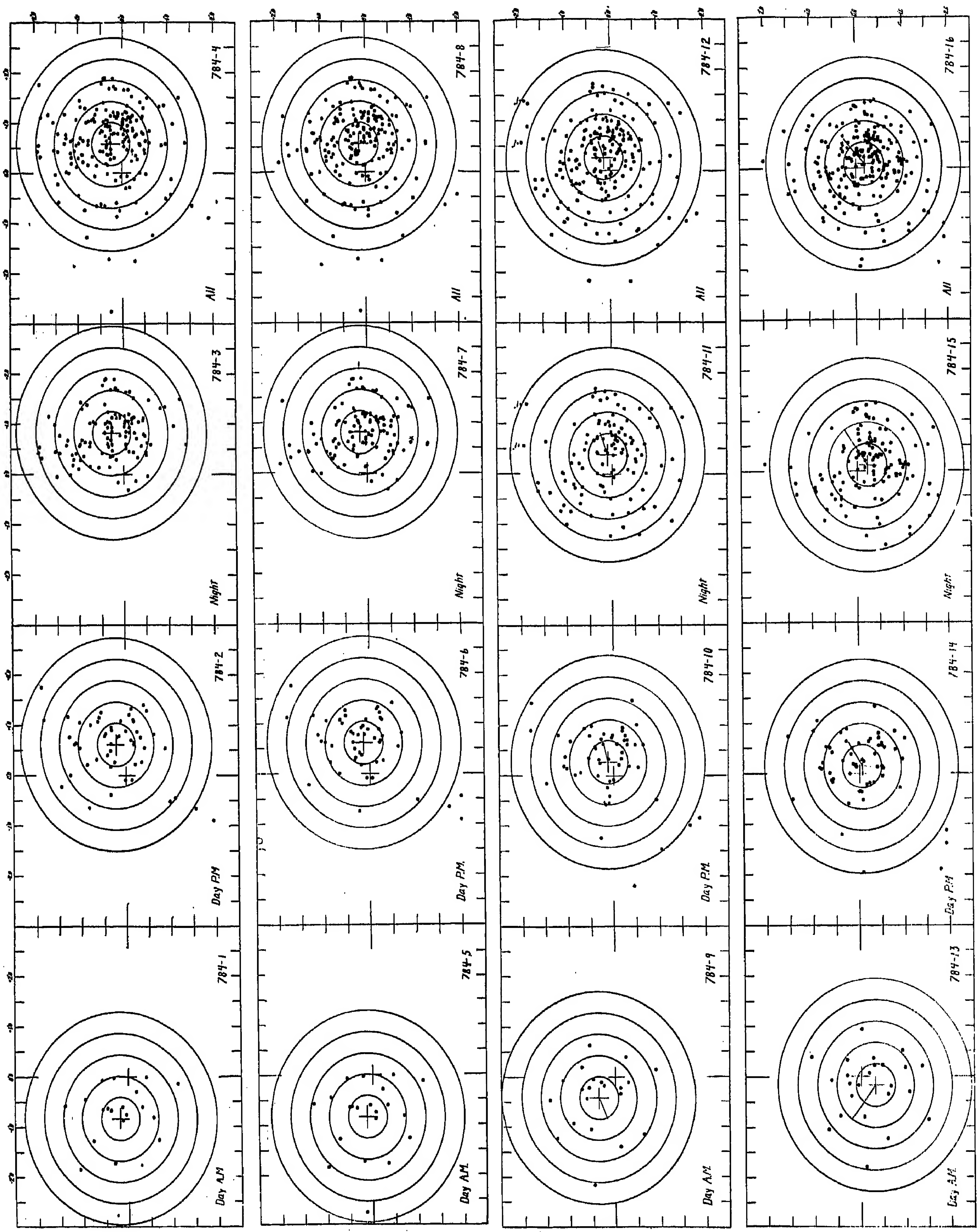

784-4
All
784-8
All
784-12
All
784-16
All
784-3
Night
784-7
Night
784-11
Night
784-15
Night
784-2
Day P.M.
784-6
Day P.M.
784-10
Day P.M.
784-14
Day P.M.
784-1
Day A.M.
784-5
Day A.M.
784-9
Day A.M.
784-13
Day A.M.

PLATE E

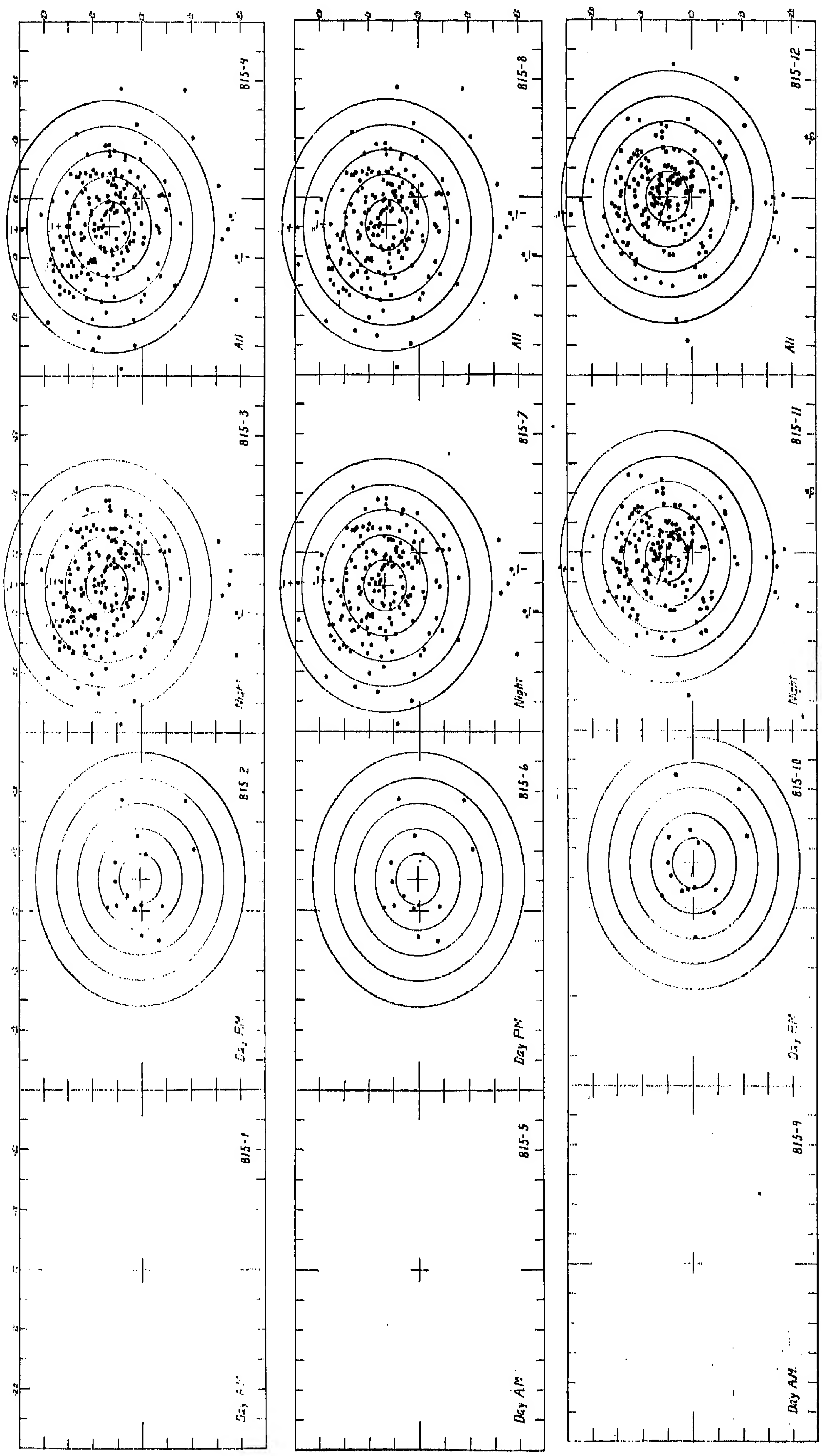
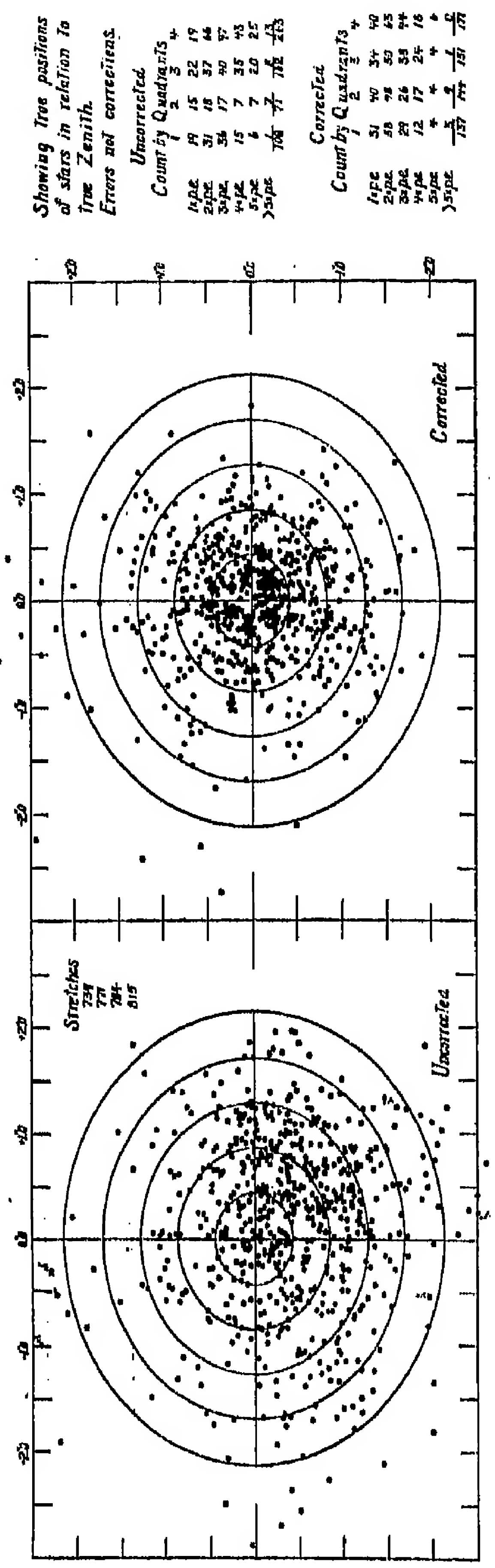


PLATE H



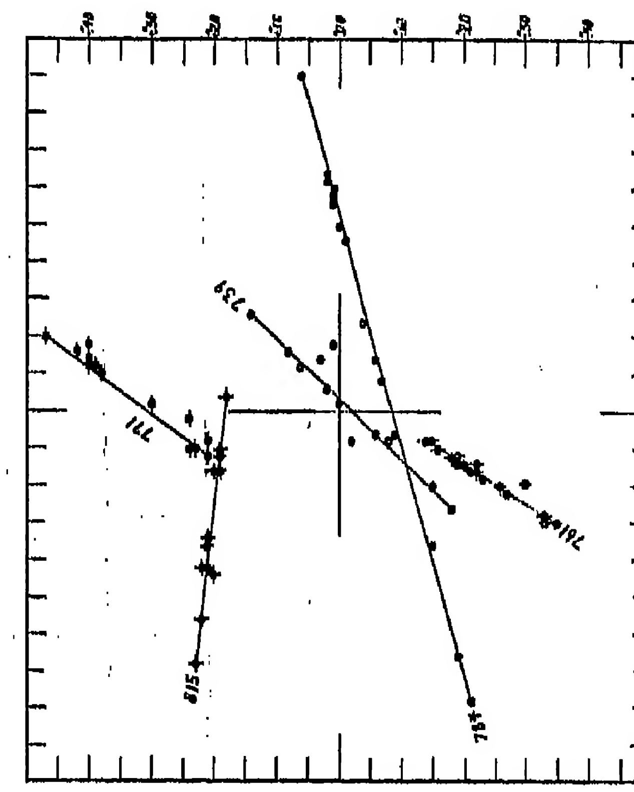
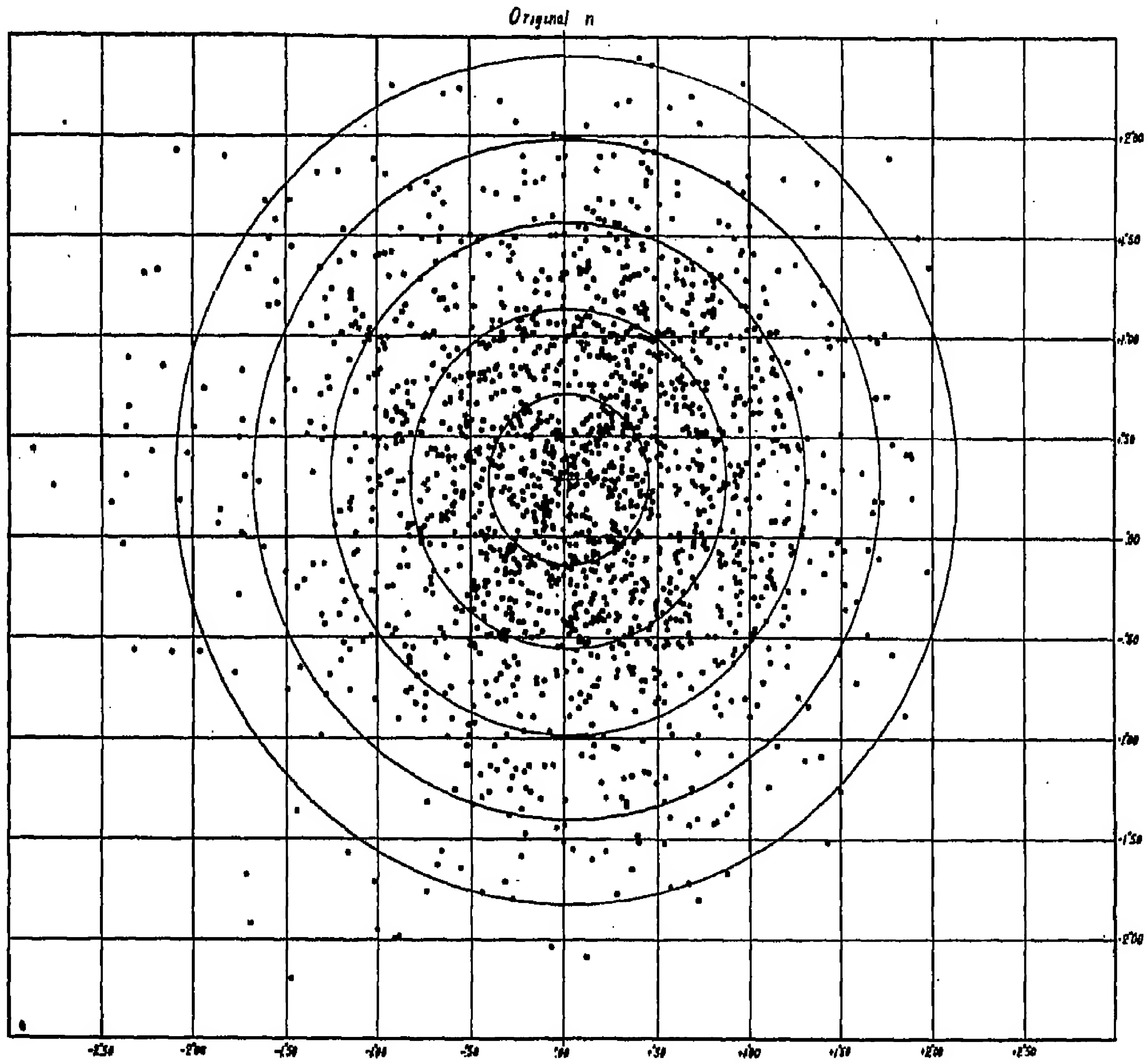


PLATE G

PLATE F



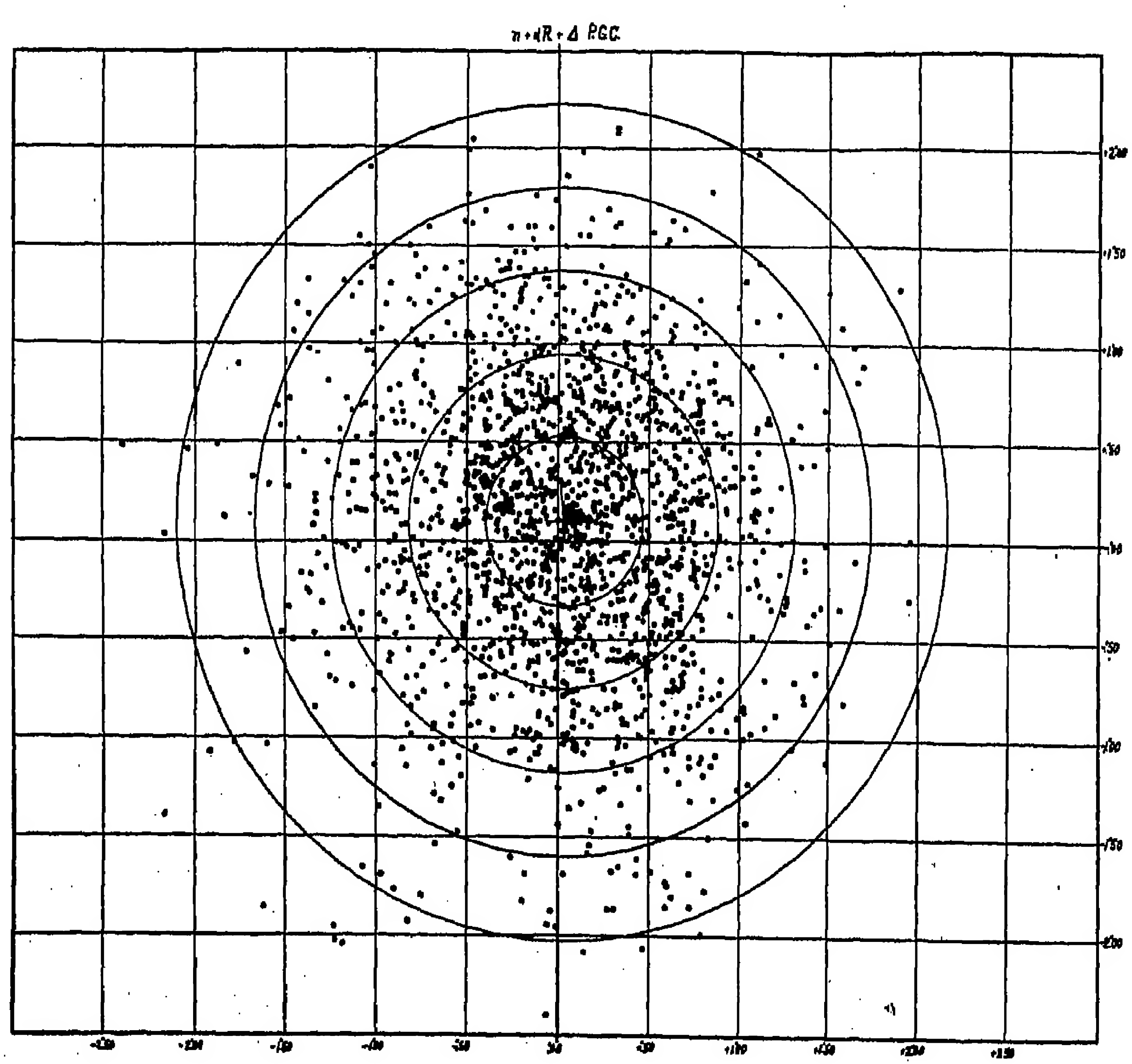


TABLE F NORTH and SOUTH STARS

Obs.	n	$n+CR$	$n+dR$	(1)	(2)	(3)	(4)
	″	″	″	″	″	″	″
84	+ 55.82	+ 44.53	− 0.17	+ 0.93	− 4.53	−44.70	− 5.46
60	+ 32.75	+ 29.74	+11.25	− 1.03	+ 3.16	−18.49	− 2.13
122	− 2.15	− 2.60	+ 8.81	− 3.73	+ 0.46	+11.41	+ 4.19
127	− 29.66	− 30.53	+ 9.13	− 1.49	+ 0.27	+39.66	+ 1.76
122	+ 54.24	+ 46.48	+17.82	− 0.69	− 2.36	−28.66	− 1.07
190	+ 22.02	+ 26.24	+ 1.26	+ 0.52	+ 1.70	−24.98	+ 1.18
200	− 45.96	− 43.85	−11.68	− 0.08	− 2.70	+32.17	− 2.62
106	+103.22	+ 59.50	+25.67	−33.75	−38.21	−33.83	− 4.46
67	+ 4.02	+ 13.91	+13.40	− 1.51	− 2.08	− 0.51	− 0.57
104	+ 33.32	+ 26.83	+12.28	− 4.85	− 9.86	−14.55	− 5.01
183	+ 43.57	+ 34.81	+ 2.94	− 2.03	+ 8.05	−31.87	+10.08
90	− 33.26	− 28.11	+10.66	− 3.49	+ 0.68	+38.77	+ 4.17
161	+132.71	+ 78.02	+ 0.90	−32.56	−42.25	−77.12	− 9.69
202	+ 39.53	+ 6.64	+ 7.74	− 6.59	− 3.75	+ 1.10	+ 2.84
221	+142.18	+117.81	+27.51	− 2.82	−21.55	−90.30	−18.73
181	− 9.90	− 4.13	− 5.82	− 0.47	− 1.40	− 1.69	− 0.93
91	+ 16.73	+ 9.54	+ 8.45	− 1.91	− 3.22	− 1.09	− 1.31
96	+ 8.44	+ 11.84	+ 8.80	− 0.79	− 0.01	− 3.04	+ 0.78
123	+ 43.85	+ 34.36	+13.73	− 0.07	+ 1.73	−20.63	+ 1.80
68	+ 43.06	+ 21.23	+ 6.04	−14.45	−16.84	−15.19	− 2.39
103	+ 3.28	+ 5.16	+ 6.12	− 0.34	+ 0.05	+ 0.96	+ 0.40
101	− 24.71	− 18.71	+ 0.56	− 4.46	− 4.37	+19.27	+ 0.09
2802	+633.1	+438.7	+175.4	−115.7	−143.4	−263.3	−27.7
Means	+ .23	+ .16	+ .06	− .04	− .06	− .09	− .01

TABLE G

S.Z.D. (AE)	Obs.	n	$n+CR$	$n+dR$	(1)	(2)	(3)	(4)
°		″	″	″	″	″	″	″
282	29	+1.04	+1.82	+ .07	+.64	+.51	−1.74	−.13
287	21	+ .05	+ .61	− .10	+.22	+.26	− .71	+.03
295	41	+ .21	+ .38	+ .13	+.12	+.02	− .25	−.09
305	125	+ .08	+ .25	+ .03	+.06	−.06	− .22	−.12
315	198	+ .08	+ .23	+ .07	+.08	−.00	− .16	−.08
325	115	− .02	+ .05	− .03	+.03	−.03	− .08	−.06
335	105	− .10	− .03	− .08	+.05	−.06	− .05	−.11
345	113	− .02	+ .03	− .03	+.02	−.03	− .06	−.05
355	115	+ .19	+ .21	+ .17	+.01	−.05	− .04	−.06
5	102	+ .23	+ .21	+ .17	−.02	−.07	− .04	−.05
15	287	+ .36	+ .31	+ .29	−.02	−.05	− .02	−.02
25	290	+ .43	+ .36	+ .33	−.05	−.07	− .02	−.01
35	408	+ .28	+ .17	+ .16	−.08	−.11	− .01	−.02
45	205	+ .29	+ .14	+ .13	−.13	−.14	− .01	−.01
55	296	+ .36	+ .12	+ .06	−.17	−.19	− .06	−.02
65	221	+ .16	− .10	− .18	−.18	−.18	− .09	+.00
73	71	+ .16	− .25	− .44	−.09	+.13	− .19	+.22
78	70	− .27	− .84	−1.20	+.08	+.92	− .35	+.84

(1) gives amount by which CR increases or decreases the original n numerically; plus means p.e. is increased and minus the p.e. is decreased. (2) gives same numerical increase or decrease when dR is applied. (3) gives $dR - CR = dR_1$. (4) gives numerical increase or decrease caused by application of dR. These apply to Table F as well as G. Values given in F are sums while those given in G are means.

sits alone with that obtained through the comparison with *P. G. C.*

	Eq.	Eq+CR	Eq + dR	Eq + dR + $\Delta P.G.C.$
Double Transits	12.69	12.51	12.67	(12.67)
P. G. C. N. of Zen.	12.59	12.45	12.64	12.64
P. G. C. S. of Zen.	12.39	12.54	12.59	12.60
P. G. C. N. and S.	12.45	12.50	12.61	12.61
	″	″	″	
North-South	+0.20	−0.09	+0.05	
	±0.31	±0.19	±0.07	

That is, the application of dR has brought the observations north and south of the zenith to agree within 0″.05, with a p. e. for a single stretch of ±0″.07. At the same time the mean equator point as derived through *P. G. C.* has been made to agree with that derived from double transits within 0″.06. Thus far we have shown that, even with an imperfect knowledge of the meteorology involved, the application of a correction for differential refraction removes the diurnal term, the systematic errors $\Delta\alpha_\alpha$, $\Delta\delta_\alpha$, $(U-L)$, and $(N-S)$, and materially decreases $\Delta\alpha_\delta$ and $\Delta\delta_\delta$; in other words it has straightened the meridian and improved the equator point.

To present the foregoing discussion in visual form we cease to consider the transits and zenith-distances as separate observations and independent of each other. The residuals formed from them may be taken as the two rectangular coördinates of the star's displacement by atmospheric effects plus accidental error. We will take for the sine component the n in Z. D., and for the cosine component the n of the transits, and we will consider the zenith of our instrument as the zero point for each coördinate. This will conform to Fig. 1. To show that the prismatic effect is real and measurable, and while systematic for a given stretch is not the same for each stretch, Plates B, C, D, E have been prepared. The separate squares of these four plates are numbered alike. 1-4 gives the position of each star as given by the original n, 5-8 gives the position when corrected for CR only, 9-12 the position when corrected for $CR + dR$. For 784 there is also exhibited the results obtained by using the finally concluded value of ρ:

$$\rho = a \sin(\alpha - \odot) + b \cos(\alpha - \odot) + c \sin 2(\alpha - \odot) + d \cos 2(\alpha - \odot)$$

Plate F contains all the stars for the 22 stretches em-

ployed, *omitting none*. On plate G are plotted the ephemerides of the meteorological zenith for five stretches for the times when observing was going on. These values are also given in tabular form in Table I. These plates should satisfy the most skeptical that dR is a positive measurable phenomenon.

The diagrams also give the opportunity to study the probable error. In the formation of *P. G. C.* Dr. Boss established as his unit for weight 1.0 the value $\pm 0''.30$ in either coördinate. Hence, if we are allowed $\pm 0''.30$ in R. A. and $\pm 0''.30$ in Z. D. it follows that if we draw a circle whose radius is $\pm 0''.30 \times 1.414$ we include the maximum deviation from the true zenith that is allowable for weight 1.0. That is, the radius of the limiting circle for unit p. e. is $\pm 0''.424$. The circles drawn represent 1–5 times the p. e. for weight 1.0. It certainly is evident that we err if we fail to take into consideration the time of day that the observation was made when discussing such a series of observations as the Albany observations. In 784 to throw out all the observations in R. A. that give n larger than $-1''.50$ among the morning observations would be unjustified, just as to throw away all the daytime observations and confine the reductions to the night observations alone is unjustified, for the night observations are nearly as far from the true meridian, only on the other side. Each set of observations, whether made morning, afternoon, or night is perfectly consistent with the zenith point derived for that time of day. Here we find perfect accordance with, and most striking confirmation of what is pictured in Fig. 1. Studying these plates in connection with Plate G we can see the combined effect of drift and diurnal term. In 771 we have an effect more or less at right angles to that of 784. Here the effect is principally in Z. D. Then in 815 we have an effect that is between the two in Z. D. but opposite in direction to 784 in R. A.

If dR is such an important term in the reduction of the Albany observations it must be the source of the major part of the before and after sunset and sunrise effect as found at other observatories. Tucker found the diurnal effect very marked at Lick Observatory. The dR term is undoubtedly the source of his trouble as given in L. O. Bulletins Nos. 292, 308 and 330. To show this his sunset-sunrise as given in No. 330 was arranged according to Mt. Hamilton Mean Time and ρ was formed from San Francisco Meteorology for 1901–2 as given in U. S. Weather Bureau Reports. (See Table J). Mean values have been used for the months in which the observations were made. These values of ρ are given together with values of sunset-sunrise. Solving for ρ we obtain $O = +0^s.1878\ \rho$ which gives column C when expanded and O–C when

TABLE I MET. ZEN. EPHEMERIDES

M. T.	739		761		771		784		815	
	Z.D.	R.A.	Z.D.	R.A.	Z.D.	R.A.	Z.D.	R.A.	Z.D.	R.A.
h	"	"	"	"	"	"	"	"	"	"
0.5	...	...	−.26	−.10	...	...	...	...	...	...
1.5	...	...	−.23	−.09	+.24	−.01	...	...	+.19	−.05
2.5	...	...	−.20	−.07	+.21	−.04	−.06	+.07	+.18	+.02
3.5	+.03	+.07	−.21	−.08	+.30	+.01	−.09	−.03	+.19	−.08
4.5	+.14	+.13	−.18	−.06	+.40	+.07	−.04	+.12	+.20	−.22
5.5	+.08	+.08	−.19	−.06	+.40	+.09	−.01	+.23	...	...
6.5	+.06	+.06	−.16	−.05	+.42	+.08	+.01	+.28	+.22	−.31
7.5	+.08	+.08	−.19	−.07	+.47	+.10	+.02	+.31	+.23	−.34
8.5	+.02	+.03	−.16	−.05	+.42	+.08	+.01	+.30	+.22	−.28
9.5	.00	+.01	−.14	−.04	+.39	+.06	+.01	+.29	+.21	−.21
10.5	+.01	+.09	−.15	−.04	+.38	+.05	.00	+.25	+.21	−.18
11.5	−.02	−.04	−.22	−.08	+.40	+.06	+.01	+.28	+.21	−.17
12.5	...	...	−.20	−.07	+.30	+.01	+.02	+.32	+.21	−.17
13.5	...	...	...	...	...	...	+.06	+.45	+.21	−.17
14.5	...	...	...	...	...	...	...	...	+.20	−.08
15.5	...	...	...	...	...	...	...	...	+.19	−.06
16.5	...	...	...	...	...	...	...	...	...	...
17.5	...	...	−.22	−.07	...	...	...	...	...	...
18.5	...	...	−.27	−.07	...	...	−.07	+.04	...	...
19.5	...	...	−.33	−.11	...	...	−.21	−.39	...	...
20.5	−.06	−.03	−.33	−.14	+.24	−.05	−.19	−.33	...	...
21.5	−.08	−.04	−.33	−.15	+.23	−.05	−.15	−.18	...	...
22.5	−.15	−.10	−.35	−.15	+.21	−.06	−...	...	...	...
23.5	−.18	−.13	−.30	−.13	...	...	...	...	...	...

applied. It is to be noted that O and C agree as to sign for 47 out of the 58 values. In column O–C, we have 9 minus and 20 plus residuals in place of 29 plus, and 11 plus and 18 minus residuals in place of 29 minus. And if we treat columns O and O–C as residuals for p. e. the ρ-term has reduced the p. e. from $\pm 0^s.030$ to $\pm 0^s.023$. If these observations could have been treated in the same manner in which 784 of the Albany observations was treated there is little doubt that the diurnal term in these residuals could have been almost completely eliminated. This rough test of the Mt. Hamilton results is interesting as evidence that the phenomenon is not purely local.

In Tables K are exhibited groups derived from the residuals as given in Second Series of Washington Observations, Vol. IX, Part 1, pages A73–381. These being the only series of observations conveniently available for testing the theory of dR the residuals as published have been examined in detail. From meteorology furnished by the U. S. Weather Bureau, μ and $\mu\rho$ were formed for each residual and μ, $\mu\rho$ and n were arranged according to Washington Mean Time. These means are exhibited in Tables K, for each group separately and for the two groups combined. In the solution for $F\mu + F\mu\rho$ each observation was assigned weight 1.0. Having in a previous test discovered the importance of the ρ-term and not wishing, at the

TABLE J

LICK — Before and After Sunset and Sunrise

M. T.	ρ	n O	n C	O − C	M. T.	ρ	n O	n C	O − C
h		s	s	s	h		s	s	s
2.7	−.138	+.051	−.026	+.077	12.6	−.052	−.003	−.010	+.007
3.2	−.048	+.051	− 9	+ 60	14.9	−.030	+.008	− 6	+ 14
3.7	+0.28	+.024	+ 5	+ 19	15.2	−.077	−.062	− 14	− 48
3.9	+.104	+.035	+ 20	+ 15	15.5	−.134	+.004	− 25	+ 29
4.2	+.050	+.013	+ 9	+ 4	15.6	−.077	−.061	− 14	− 47
4.3	+.074	+.101	+ 14	+ 87	15.9	−.146	−.026	− 27	+ 1
4.3	+.094	+.012	+ 18	− 6	16.0	−.139	−.011	− 26	+ 15
4.7	+.070	+.075	+ 14	+ 61	16.3	−.238	−.070	− 43	− 27
4.7	+.014	+.049	+ 3	+ 46	16.5	−.224	−.017	− 42	+ 25
4.9	+.048	+.047	+ 9	+ 38	16.5	−.254	−.032	− 48	+ 16
5.3	+.108	+.031	+ 20	+ 11	16.8	−.143	−.055	− 27	− 28
5.4	+.084	+0.59	+ 16	+ 43	16.9	−.098	−.020	− 18	− 2
5.4	+.113	+.069	+ 21	+ 48	17.0	−.099	−.040	− 19	− 21
5.8	+.125	+.053	+ 24	+ 29	17.3	+.042	−.023	+ 8	− 31
5.9	−.002	+.037	0	+ 37	17.4	+.139	−.052	+ 26	− 78
6.0	+.091	+.028	+ 17	+ 11	17.6	−.036	−.036	− 7	− 29
6.1	+.204	+.077	+ 38	+ 39	17.7	+.066	−.029	+ 12	− 41
6.3	+.190	+.049	+ 36	+ 13	17.8	+.048	−.050	+ 9	− 59
6.3	−.110	+.023	− 21	+ 44	18.0	+.006	−.020	+ 1	− 21
6.4	+.063	+.001	+ 12	− 11	18.2	−.136	−.058	− 26	− 32
6.5	+.196	+.027	+ 37	− 10	18.2	+.036	−.006	+ 7	− 13
6.8	+.183	+.021	+ 34	− 13	18.4	−.098	−.029	− 18	− 11
6.9	+.084	+.001	+ 16	− 15	18.7	−.302	−.067	− 57	− 10
7.0	+.266	+.019	+ 50	− 31	18.7	−.026	−.026	− 5	− 21
7.3	+.112	+.009	+ 21	− 12	18.9	−.244	−.024	− 46	+ 22
7.6	+.310	+.049	+ 58	− 9	19.3	−.122	−.020	− 23	+ 3
8.0	+.206	+.009	+ 39	− 30	19.4	−.394	−.067	− 74	+ 7
8.4	+.092	+.043	+ 17	+ 26	19.7	−.174	−.047	− 33	− 14
8.9	+.036	+.006	+ 2	+ 4	20.0	−.204	−.008	− 38	+ 30

present state of the work, to revise including that term, a solution was made for a sin MT + cos MT term as this form of expression has been shown to well represent the ρ. For the dR term we obtained for the 5^h group

$$+0^s.0036\ F\mu \quad +0^s.0056\ F\mu\rho$$
$$+0''.373\ F\mu \quad +0''.051\ F\mu\rho$$

For the 18^h group

$$+0^s.0119\ F\mu \quad +0^s.0034\ F\mu\rho$$
$$+0''.218\ F\mu \quad +0''.101\ F\mu\rho$$

For 5^h and 18^h groups combined

$$+0^s.0077\ F\mu \quad +0^s.0057\ F\mu\rho$$
$$+0''.294\ F\mu \quad +0''.059\ F\mu\rho$$

These were expanded for each group as exhibited in the column headed dR. Subtracting these from Mean n_α and n_δ, the column $(O-C)_1$ was obtained. Each value of $(O-C)_1$ shows a well marked cosine term. Also, it is to be noted that the application of dR has almost completely eliminated the constants in R. A. and Decl. From the above tables we have

TABLE K R. A. Group at $5^h\ 42^m$

M. T.	n_α	dR	$(O-C)_1$	ρ	$(O-C)_2$
h	s	s	s	s	s
3.6	−.043	+.010	−.053	−.019	−.034
4.6	−.041	+.012	−.053	−.016	−.037
5.6	−.011	+.014	−.025	−.012	−.013
6.5	+.015	+.016	−.001	−.008	+.007
7.4	+.019	+.013	+.006	−.003	+.009
8.5	+.019	+.013	+.006	+.003	+.003
9.5	+.028	+.013	+.015	+.008	+.007
10.4	+.018	+.011	+.007	+.013	−.006
11.6	+.017	+.011	+.006	+.017	−.011
12.5	+.019	+.010	+.009	+.020	−.011
13.5	+.016	+.011	+.005	+.021	−.016
14.5	+.028	+.011	+.017	+.021	−.004
15.5	+.028	+.011	+.017	+.019	−.002
16.5	+.016	+.011	+.005	+.016	−.011
17.5	+.020	+.010	+.010	+.013	−.003
18.4	−.012	+.007	−.019	+.008	−.027
19.2	+.002	+.005	−.003	+.004	−.007
Means	+.010	+.011	−.003		−.009

TABLE K Decl. Group at $5^h\ 42^m$

μ	$\mu\rho$	M. T.	n_δ	dR	$(O-C)_1$	ρ	$(O-C)_2$
		h	″	″	″	″	″
0.972	+.032	3.6	+.10	+.60	−.50	−.22	−.28
0.985	+.162	4.6	+.08	+.63	−.55	−.17	−.38
1.000	+.356	5.6	+.40	+.65	−.25	−.11	−.14
1.006	+.483	6.5	+.76	+.66	+.10	−.05	+.15
1.030	+.263	7.4	+.76	+.66	+.10	+.02	+.08
1.040	+.242	8.5	+.84	+.67	+.17	+.09	+.08
1.042	+.220	9.5	+.89	+.66	+.23	+.15	+.08
1.044	+.118	10.4	+.79	+.66	+.13	+.20	−.07
1.051	+.103	11.6	+.76	+.66	+.10	+.24	−.14
1.050	+.026	12.5	+.90	+.65	+.25	+.26	−.01
1.030	+.064	13.5	+.85	+.64	+.21	+.26	−.05
1.024	+.090	14.5	+.71	+.64	+.07	+.25	−.18
1.011	+.103	15.5	+.75	+.64	+.11	+.22	−.11
1.009	+.144	16.5	+.92	+.64	+.28	+.17	+.11
0.992	+.079	17.5	+.61	+.62	−.01	+.12	−.13
0.988	−.115	18.4	+.50	+.56	−.06	+.06	−.12
0.980	−.277	19.2	+.61	+.59	+.02	.00	+.02
			+.68	+.64			−.06

	R. A. Mean n	R. A. Mean dR	R. A. Mean $(O-C)_1$	Decl. Mean n	Decl. Mean dR	Decl. Mean $(O-C)_1$
	s	s	s	″	″	″
5^h	+0.010	+0.011	−0.003	+0.68	+0.64	+0.02
18^h	+0.033	+0.033	+0.002	+0.46	+0.42	+0.04
All	+0.024	+0.022	+0.002	+0.51	+0.52	−0.01

TABLE K R.A. Group at 18^h 5^m

M. T.	n_α	dR	(O–C)$_1$	ρ	(O–C)$_2$
h	s	s	s	s	s
2.7	+.010	+.031	–.021	–.015	–.006
3.6	+.009	+.032	–.023	–.014	–.009
4.5	+.003	+.034	–.031	–.013	–.018
5.5	+.026	+.035	–.009	–.011	+.002
6.5	+.031	+.036	–.005	–.008	+.003
7.5	+.041	+.034	+.007	–.005	+.012
8.5	+.037	+.034	+.003	–.001	+.004
9.5	+.034	+.033	+.001	+.003	–.002
10.4	+.047	+.033	+.014	+.006	+.008
11.4	+.045	+.032	+.013	+.010	+.003
12.5	+.048	+.032	+.016	+.013	+.003
13.5	+.039	+.032	+.007	+.014	–.007
14.5	+.042	+.034	+.008	+.015	–.007
15.5	+.038	+.033	+.005	+.015	–.010
16.5	+.045	+.034	+.011	+.013	–.002
17.5	+.043	+.034	+.009	+.011	–.002
18.5	+.045	+.033	+.012	+.008	+.004
19.4	+.023	+.032	–.009	+.005	–.014
20.3	+.037	+.031	+.006	+.002	+.004
21.1	+.055	+.032	+.023	–.002	+.025
Means	+.033	+.033	+.002		.000

TABLE K R. A. Groups 5^h 42^m and 18^h 5^m

M. T.	n_α	dR	(O–C)$_1$	ρ	(O–C)$_2$
h	s	s	s	s	s
2.7	+.010	+.019	–.009	–.013	+.004
3.6	–.009	+.021	–.030	–.014	–.016
4.5	–.018	+.023	–.041	–.014	–.027
5.6	+.007	+.026	–.019	–.013	–.006
6.5	+.023	+.028	–.005	–.012	+.007
7.5	+.030	+.025	+.005	–.009	+.014
8.5	+.028	+.025	+.003	–.006	+.009
9.5	+.031	+.024	+.007	–.002	+.009
10.4	+.032	+.023	+.009	+.001	+.008
11.5	+.028	+.022	+.006	+.005	+.001
12.5	+.029	+.022	+.007	+.008	–.001
13.5	+.024	+.021	+.003	+.011	–.008
14.5	+.033	+.022	+.011	+.013	–.002
15.5	+.032	+.022	+.010	+.014	–.004
16.5	+.031	+.023	+.008	+.014	–.006
17.5	+.032	+.022	+.010	+.013	–.003
18.5	+.019	+.019	.000	+.012	–.012
19.4	+.016	+.018	–.002	+.009	–.011
20.3	+.037	+.018	+.019	+.006	+.013
21.6	+.055	+.017	+.038	+.004	+.034
Means	+.024	+.022	+.002		.000

TABLE K Decl. Group at 18^h 5^m

μ	$\mu\rho$	M. T.	n_δ	dR	(O–C)$_1$	ρ	(O–C)$_2$
		h	"	"	"	"	"
0.995	–.049	2.7	–.05	+.37	–.42	–.36	–.06
0.983	+.109	3.6	–.05	+.40	–.45	–.31	–.14
0.978	+.310	4.5	+.11	+.44	–.33	–.25	–.08
0.971	+.522	5.5	+.40	+.47	–.07	–.16	+.09
0.967	+.559	6.5	+.47	+.48	–.01	–.06	+.05
0.965	+.398	7.5	+.47	+.45	+.02	+.04	–.02
0.963	+.370	8.5	+.53	+.44	+.09	+.14	–.05
0.961	+.262	9.5	+.59	+.42	+.17	+.22	–.05
0.962	+.226	10.4	+.65	+.42	+.23	+.29	–.06
0.969	+.161	11.4	+.68	+.40	+.28	+.35	–.07
0.977	+.146	12.5	+.77	+.40	+.37	+.38	–.01
0.988	+.126	13.5	+.81	+.41	+.41	+.39	+.02
0.907	+.154	14.5	+.72	+.41	+.31	+.36	–.05
1.004	+.159	15.5	+.59	+.43	+.16	+.32	–.16
1.013	+.134	16.5	+.74	+.43	+.31	+.25	+.06
1.026	+.143	17.5	+.61	+.42	+.19	+.16	+.03
1.035	–.017	18.5	+.48	+.39	+.09	+.06	+.03
1.044	–.144	19.4	+.35	+.38	–.03	–.03	.00
1.048	–.263	20.3	+.26	+.37	–.11	–.12	+.01
1.044	–.376	21.1	–.05	+.38	–.43	–.19	–.24
Means			+.46	+.42	+.04		–.04

TABLE K Decl. Groups 5^h 42^m and 18^h 5^m

μ	$\mu\rho$	M. T.	n_δ	dR	(O–C)$_1$	ρ	(O–C)$_2$
		h	"	"	"	"	"
0.995	–.069	2.7	–.05	+.50	–.55	–.36	–.19
0.980	+.083	3.6	.00	+.51	–.51	–.30	–.21
0.981	+.239	4.5	+.10	+.52	–.42	–.23	–.19
0.986	+.438	5.6	+.40	+.54	–.14	–.13	–.01
0.987	+.521	6.5	+.62	+.55	+.07	–.04	+.11
0.998	+.331	7.5	+.61	+.54	+.07	+.07	.00
1.002	+.306	8.5	+.68	+.54	+.14	+.18	–.04
1.001	+.241	9.5	+.74	+.53	+.21	+.26	–.05
1.005	+.169	10.4	+.72	+.53	+.19	+.33	–.14
1.021	+.125	11.5	+.73	+.52	+.21	+.38	–.17
1.026	+.066	12.5	+.86	+.52	+.34	+.41	–.07
1.016	+.085	13.5	+.84	+.52	+.32	+.40	–.08
1.015	+.111	14.5	+.71	+.52	+.19	+.37	–.18
1.008	+.127	15.5	+.68	+.52	+.16	+.31	–.15
1.011	+.139	16.5	+.83	+.53	+.30	+.23	+.07
1.009	+.111	17.5	+.61	+.52	+.09	+.14	–.05
1.014	–.062	18.5	+.49	+.49	.00	+.04	–.04
1.022	–.188	19.4	+.44	+.50	–.06	–.06	.00
1.048	–.263	20.3	+.26	+.54	–.28	–.15	–.13
1.044	–.376	21.1	–.05	+.56	–.61	–.23	–.38
Means			+.51	+.52	–.01		–.10

The small values of mean $(O-C)_1$ show that dR has straightened and shifted the meteorological meridian and zenith to agree with the true meridian and zenith of the instrument, and in doing so has eliminated the resultant drift of the observations toward the northeast. $(O-C)_1$ was then solved for a sin WMT $+ b$ cos WMT to obtain the value of the ρ-term in the residual diurnal effect, as for Albany, giving

5^h group

$\rho = -0^s.0101$ sin W.M.T. $-0^s.0184$ cos W.M.T.
$= +0^s.0210$ cos ($13^h\ 55^m$ − W.M.T.)
$= -0''.082$ sin W.M.T. $-0''.253$ cos W.M.T.
$= +0''.265$ cos ($13^h\ 12^m$ − W.M.T.)

18^h group

$\rho = -0^s.0096$ sin W.M.T. $-0^s.0114$ cos W.M.T.
$= +0^s.0149$ cos ($14^h\ 40^m$ − W.M.T.)
$= -0''.103$ sin W.M.T. $-0''.373$ cos W.M.T.
$= +0''.387$ cos ($13^h\ 2^m$ − W.M.T.)

All

$\rho = -0^s.0125$ sin W.M.T. $-0^s.0070$ W.M.T.
$= +0^s.0142$ cos ($16^h\ 3^m$ − W.M.T.)
$= -0''.085$ sin W.M.T. $-0''.398$ cos W.M.T.
$= +0''.407$ cos ($12^h\ 48^m$ − W.M.T.)

The results of these solutions for $F\mu + F\mu\rho$ and ρ terms show that the normal shift for Washington, due to dR, was

$+0^s.022$ $+0^s.0142$ cos ($16^h\ 3^m$ − W.M.T.)
$+0''.52$ $+0''.407$ cos ($12^h\ 48^m$ − W.M.T.)

Objection may be made to removing the constant by means of $F\mu + F\mu\rho$ instead of merely taking out a constant. The taking out of the constant before all known forms of error have been corrected for is not defensible, especially when such a well marked cosine term is contained in the residuals. If the n_α and n_δ were known for each hour of the 24 hours we could assume the means were true constants, but to do so in the present case would be erroneous. In order to show that the ρ-term (a sin W.M.T. $+ b$ cos W.M.T.) will be very little affected by the removal of a constant, a constant was applied to each group and, solving for a sine and and cosine term, we obtained

5^h group
$+0^s.010$ $-0^s.0075$ sin W.M.T. $-0^s.0209$ cos W.M.T.
$+0^s.010$ $+0^s.0222$ cos ($13^h\ 19^m$ − W.M.T.)
$+0''.68$ $-0''.076$ sin W.M.T. $-0''.242$ cos W.M.T.
$+0''.68$ $+0''.254$ cos ($13^h\ 10^m$ − W.M.T.)

18^h group
$+0^s.033$ $-0^s.0078$ sin W.M.T. $-0^s.0138$ cos W.M.T.
$+0^s.033$ $+0^s.0159$ cos ($13^h\ 58^m$ − W.M.T.)
$+0''.46$ $-0''.093$ sin W.M.T. $-0''.365$ cos W.M.T.
$+0''.46$ $+0''.377$ cos ($12^h\ 58^m$ − W.M.T.)

All
$+0^s.024$ $-0^s.0102$ sin W.M.T. $-0^s.0080$ cos W.M.T.
$+0^s.024$ $+0^s.0130$ cos ($15^h\ 30^m$ − W.M.T.)
$+0''.51$ $-0''.068$ sin W.M.T. $-0''.405$ W.M.T.
$+0''.51$ $+0''.411$ cos ($12^h\ 38^m$ − W.M.T.)

The almost perfect agreement of these values with those obtained by using $F\mu + F\mu\rho$ would indicate that the application of dR would help remove the large term in the Washington observations discussed. Also, it will be noticed that the addition of the terms sin 2 MT + cos 2 MT, which were not used in this investigation would, as for Albany, improve the results. That n_δ is not a constant but a very strong, well marked diurnal term is shown in the following exhibit.

WMT	Cos′MT	n'_δ	WMT	Cos″MT	n''_δ	Cos′ −Cos″	n'_δ $-n''_\delta$
h m			h m				
2 41	+.763	−0.05	14 30	−.793	+0.71	+1.56	−0.76
3 38	+.581	0.00	15 30	−.609	+0.68	+1.19	−0.68
4 32	+.375	+0.10	16 29	−.387	+0.83	+0.76	−0.73
5 33	+.118	+0.40	17 28	−.139	+0.61	−0.02	−0.21
6 28	−.122	+0.62	18 28	+.122	+0.49	−0.24	+0.13
7 27	−.371	+0.61	19 22	+.350	+0.44	−0.72	+0.17
8 30	−.609	+0.68	20 17	+.570	+0.26	−1.17	+0.42
9 28	−.788	+0.74	21 6	+.725	−0.05	−1.51	+0.79

In the table, the dependence of n_δ upon the cosine W.M.T. will be recognized at a glance.

This marked dependence upon the time of day that the observation was made is further shown by the variations in the 1700 observations of α *Lyræ*. (See *Popular Astronomy*, Vol. XXX, No. 3, page 165). It would be extremely interesting to see if the correction to nutation from night observations and from day observations would not be much more accordant if the observations were corrected for the effects of dR. And this confirmation of the diurnal term and dR is especially valuable as the observations were made with the prime vertical transit instrument.

In the same volume appears another article which also shows that a diurnal effect is being noticed in *Sun* observations.

	$\sec^2 z$	Δ_δ	Δ_δ comp.		$\sec^2 z$	Δ_δ	Δ_δ comp.
Jan.	+2.41	+1.01	+1.01	July	−0.53	−0.89	−0.22
Feb.	+0.95	+1.19	+0.40	Aug.	−0.43	−0.83	−0.18
Mar.	+0.11	+0.18	+0.05	Sept.	−0.14	−0.69	−0.06
Apr.	−0.34	−0.37	−0.14	Oct.	+0.52	+0.25	+0.22
May	−0.52	−0.48	−0.22	Nov.	+1.82	+0.67	+0.76
June	−0.56	−0.52	−0.24	Dec.	+3.00	+1.02	+1.26

Column $\sec^2 z$ is derived by subtracting $\sec^2 z$ of the Washington latitude from $\sec^2 z$ of the *Sun's* declination. $\Delta\delta$ is copied from HAMMOND's article and $\Delta\delta$ comp. is derived from a solution giving $\Delta\delta = +0''.42 \sec^2 z$. The perfect agreement in sign and the general reduction of the original residuals suggests that there may be some connection between dR and $\Delta\delta$.

In the *Annuaris Astronomicopel* 1923, TORINO appears an article on the Diurnal Variation of Latitude which would appear to be another manifestation of the effect of dR. After conference with DR. KIMURA on the occasion of his recent visit to the Dudley Observatory, it seems probable that the effect of differential refraction on the latitude observations will remove a large part at least of the z-term. Then there is the question as to what extent the dR term effects the observations of the *Sun*, *Moon*, and *Planets*, both directly and indirectly, through the adopted clock corrections. Also, this phenomenon is undoubtedly the cause of different systematic corrections to catalogues, depending on whether we use bright or faint stars. In the observations of the bright stars, taken more or less throughout the 24 hours of the day, the effect of dR will tend to eliminate; in the observations of the faint stars taken always at night it will not. The various attempts to solve this perplexing problem have held up the reductions of the San Luis and Albany observations for some time and we are fully aware of the impatience felt, in some quarters, over the long delay. Inasmuch, however, as the series of investigations have enabled us to explain in a natural way so many of the points which have been puzzling meridian observers for years, we feel that the time has been well spent. With physical explanations for most of the known systematic errors and means of eliminating or evaluating them, we plan to reduce and discuss all the Fundamental stretches both in R. A. and Z. D., for both series of observations, those of San Luis and Albany, as one connected series, as only by so doing can certain fundamental questions be settled.

In this preliminary investigation *no rejections nor changes in the original data have been allowed.* All corrections, such as corrections to circle readings derived from the Nadir, (N–S) both in R. A. and Z. D., and sine flexure, have been considered as absolute. In other words, *all instrumental corrections determined by special observations have been considered as final and so used.* All observations have been used with their full weight. In the second approximation we shall feel warranted in rejecting all residuals exceeding 5 x p. e., after the residuals have been corrected by the first approximation. In this way, we hope to obtain true positions of the stars and not the positions of the stars as they should have been to agree with the other determinations. And, in using the places of *P. G. C.*, we will endeavor to so combine the observations that, except for the zero point in R. A., concluded positions will be independent of *P. G. C.* As indicated elsewhere, *P. G. C.* places will be used only as a rough scale to determine the systematic errors in the observations and then the errors of the scale will be determined.

SUMMARY

1. There is a varying prismatic effect due to the changes in the strata of the atmosphere.

2. The total effect is essentially a shift of the meteorological zenith.

3. The temperature, and not the barometer, is the controlling factor.

4. Expressions have been derived for the effect of this phenomenon on observations.

5. When these expressions have been applied to the observations, the (N–D) has been substantially reduced. If humidity and *Sun*-temperature had been employed in the original solution for dR, the diurnal term would have been completely eliminated.

6. The diurnal term is due directly to the atmosphere. Its law is $a \sin (\alpha - \odot) + b \cos (\alpha - \odot) + c \sin 2 (\alpha - \odot) + d \cos 2 (\alpha - \odot)$.

7. The systematic corrections $\Delta\alpha_\alpha$ and $\Delta\delta_\alpha$ have been practically eliminated.

8. The application of dR brings the observations north and south of the zenith into better agreement without the application of a constant.

9. Tests of published results of other observatories show that the phenomenon is not local.

PART II

Method of Fundamental Reduction

Part I contains a discussion of the effect of refraction upon observations. In order that the results may not be attributed to the method of reduction, it is well to state what steps have been taken to insure freedom from systematic error in the final results through a rigorous treatment of the material, whereby the probable sources of systematic error have been evaluated and eliminated. No reduction can be called fundamental unless it can be shown by an analysis of the method used that the observations themselves fix one point in the sky free from any assumed places of the stars used; that is, a fundamental method must give a zero point in the sky. Bearing this criterion in mind, let us develop, as briefly as possible, the method used in reducing the Albany observations. Incidentally, it should be borne in mind that in the following discussions we are considering systematic and not accidental errors.

Zenith Distances

For observations in zenith distance, we must fix the zero point through the pole-height, which can be derived directly from observations of the successive transits of a circumpolar star. Once we have the zenith distance of the pole, we know the zenith distance of the equator; that is, we would have the equator-point, were it not for the several systematic errors that effect every zenith distance observation. So, if we can determine the effect of every error on the equator-point and correct our observations for them, we can get the true pole and the true equator-point.

There are five sources of error in zenith distance observations: the Circles, the Telescope, the Reticule, the Personal Equation, and the Atmosphere.

The Circles.— Corrections for the division errors and circle flexures have been very thoroughly determined for the circles of the Olcott Meridian Circle and can be taken as definitive.

The Telescope.— The sine-flexure has been very carefully determined by observations made in connection with the north and south collimators. This can be taken as definitive.

The Reticule.— Double settings and double readings in zenith distance were made at frequent intervals to determine the inclination of the fixed zenith distance wire. So that this correction can be considered as without effect on the corrected zenith distance.

Personal Equation.— Special observations were made by each observer to determine this correction of the form feet north minus feet south. These values were applied and can be considered as a very close approximation to the truth.

Atmosphere.— The usual corrections for vertical refraction were computed, based on the Pulkova Tables and our own meteorology. In-so-far as they go, they serve for a first approximation.

The circles, corrected for, can be considered nearly perfect, the telescope and reticule can also be considered as giving perfect readings except for any illumination error, and the readings can be considered as free from personal equation, except as personal focus may superimpose on illumination a secondary illumination error.

The effect of these errors will be very slight, so that when we take two observations of the same circumpolar star twelve hours apart the mean is practically free from all sources of error, except atmospheric, and can be taken as giving the zenith distance of the pole.

Since the equator is 90° from the pole, we have

$$\text{True } z \text{ of pole } +90^\circ = \text{Equator-Point } AE \text{ and } BW^*$$
$$\text{True } z \text{ of pole } -90^\circ = \text{Equator-Point } AW \text{ and } BE$$

For studying the systematic errors of observed zenith distance, we will use the *Preliminary General Catalogue* of Dr. Lewis Boss, and instead of transforming zenith distances to declinations, we will transform the declinations of *P. G. C.* to Albany zenith distances by means of the equator-points derived as indicated above.

$$\text{True } z \text{ of pole } +90^\circ - PGC\ \delta = \text{Computed Z.D. } AE \text{ and } BW$$
$$\text{True } z \text{ of pole } -90^\circ + PGC\ \delta = \text{Computed Z.D. } AW \text{ and } BE$$

We have thus introduced into our discussion the systematic errors of *P. G. C.* so that, sooner or later, we will have to evaluate $\Delta\delta_\alpha$ and $\Delta\delta_\delta$, which designate respectively the systematic error in declination dependent upon right-ascension, and the systematic error in declination dependent upon declination.

In tracing the effect of each form of systematic error,

*The circles are distinguished by the letters *A* and *B*. For economy of reference *AE* refers to readings on circle *A*, clamp east; *BW* refers to readings on circle *B*, clamp west, etc. Circle *A* is on the clamp end of the axis. *AE* gives *S. Z. D.*

we will consider that all other errors, save the one under consideration, do not exist and thus obtain the effect of each error upon the equator-point, and we will treat of corrections, not errors. As the effect of differential refraction will be the most erratic and the most important of our errors, we will examine for dR first.

Refraction.

$$\begin{aligned} \text{True } z \text{ above pole} &= z' + CR' + dR_1' \\ \text{True } z \text{ below pole} &= z'' + CR'' + dR_1'' \\ \text{True } z \text{ of pole} &= z_0 + CR_0 + (dR_1)_0 \end{aligned}$$

$$\text{Where } \frac{z'+z''}{2} = z_0; \quad \frac{CR'+CR''}{2} = CR_0; \quad \text{and } \frac{dR_1'+dR_1''}{2} = (dR_1)_0$$

AE and *BW*:

$$\begin{aligned} \text{Equator-Point} &= Eq. = z_0 + 90^\circ + CR_0 + (dR')_0 \\ \text{Computed True } z &= Eq. - \delta_0 \\ &= z_0 + 90^\circ - \delta_0 + CR_0 + (dR_1)_0 \\ \text{Observed True } z &= z' + CR' + dR_1' \end{aligned}$$

$$z_0 + 90^\circ - \delta_0 + CR_0 + (dR_1)_0 = z' + CR' + dR_1'$$
$$(z_0 + 90^\circ - \delta_0) - z' = n = CR' - CR_0 + dR_1' - (dR_1)_0$$

AW and *BE*:

$$\begin{aligned} \text{Equator-Point} &= Eq. = z_0 - 90^\circ + CR_0 + (dR_1)_0 \\ \text{Computed True } z &= Eq. + \delta_0 \\ &= z_0 - 90^\circ + \delta_0 + CR_0 + (dR_1)_0 \\ \text{Observed True } z &= z' + CR' + dR_1' \end{aligned}$$

$$(z_0 - 90^\circ + \delta_0) - z' = n = CR' - CR_0 + dR_1' - (dR_1)_0$$

As CR_0 and $(dR_1)_0$ will each be constant for the stretch, we can neglect them; then for all positions

$$n = CR' + dR_1' = CR' + e' F' \mu + f' F' \mu\rho \qquad (4)$$

These values of dR come out in the form of corrections to the zenith distances read. Hence, if we subtract these values, expanded for each star, we obtain n' free from refractional errors, so far as we know.

We have thus disposed of atmospheric errors and have left the systematic errors of *P. G. C.*, and the residual systematic errors of the observations.

As the positions of *P. G. C.* do not enter into the formation of our equator-point, we have for observed equator reading,

AE and *BW*:
$$Eq. = z_0 + 90^\circ \qquad \text{True } z = z_0 + 90^\circ - \delta_0$$

AW and *BE*:
$$Eq. = z_0 - 90^\circ \qquad \text{True } z = z_0 - 90^\circ + \delta_0$$

P. G. C. $\delta + \Delta\delta_\alpha + \Delta\delta_\delta = \delta_0$, where δ_0 = True Decl.

AE and *BW*:
$$\begin{aligned} \text{Computed True } z &= z_0 + 90^\circ - (\delta + \Delta\delta_\alpha + \Delta\delta_\delta) \\ \text{Observed True } z &= z' \end{aligned}$$

$$(z_0 + 90^\circ - \delta) - z' - \Delta\delta_\alpha - \Delta\delta_\delta = 0$$
$$(z_0 + 90^\circ - \delta) - z' = n = +\Delta\delta_\alpha + \Delta\delta_\delta$$

AW and *BE*:
$$\begin{aligned} \text{Computed True } z &= z_0 - 90^\circ + (\delta + \Delta\delta_\alpha + \Delta\delta_\delta) \\ \text{Observed True } z &= z' \end{aligned}$$

$$(z_0 - 90^\circ + \delta) - z' = n = -\Delta\delta_\alpha - \Delta\delta_\delta$$

where $\Delta\delta_\alpha = a \sin\alpha + b\cos\alpha + c\sin 2\alpha + d\cos 2\alpha$

The above equations hold good for all upper transits and cover the four positions of the instrument. For stars observed below pole, we have a different set of equations due to the manner in which we use the δ of *P. G. C.* Instead of using δ, we use

$$(180^\circ - \delta_0) \text{ or } 180^\circ - (\delta + \Delta\delta_\alpha + \Delta\delta_\delta)$$

so we have for below pole observations

AE and *BW*:
$$[z_0 + 90^\circ - (180^\circ - \delta)] - z' = n = -\Delta\delta_\alpha - \Delta\delta_\delta$$

AW and *BE*:
$$[z_0 - 90^\circ + (180^\circ - \delta)] - z' = n = +\Delta\delta_\alpha + \Delta\delta_\delta$$

From which it follows, we can write our equations in the form

$$\left.\begin{aligned} AE \quad n &= +\Delta\delta_\alpha + \Delta\delta_\delta \text{ above pole} = -\Delta\delta_\alpha - \Delta\delta_\delta \text{ below pole} \\ AW \quad n &= -\Delta\delta_\alpha - \Delta\delta_\delta \text{ above pole} = +\Delta\delta_\alpha + \Delta\delta_\delta \text{ below pole} \\ BE \quad n &= -\Delta\delta_\alpha - \Delta\delta_\delta \text{ above pole} = +\Delta\delta_\alpha + \Delta\delta_\delta \text{ below pole} \\ BW \quad n &= +\Delta\delta_\alpha + \Delta\delta_\delta \text{ above pole} = -\Delta\delta_\alpha - \Delta\delta_\delta \text{ below pole} \end{aligned}\right\} \quad (5)$$

These equations give us the key for combining the four positions of the instrument in order to evaluate $\Delta\delta_\alpha$ and $\Delta\delta_\delta$. One correction varies according to R.A., while the other varies according to zenith distance.

As experience has shown that $\Delta\delta_\delta$ is generally the larger, we will evaluate that first. And moreover, since we are using all our observations over one or more years, it follows that, when we combine for $\Delta\delta_\delta$, we eliminate the $\Delta\delta_\alpha$ to a very large extent, which is important. This cannot be said of $\Delta\delta_\delta$ when we combine to obtain $\Delta\delta_\alpha$. Beginning at the zenith, divide into equal groups north and south. A little planning will enable one to place the pole at or near one of the dividing points.

Having formed the means for these groups for each position, and collecting them we have

$$\frac{AE + BW - AW - BE}{4} = +\Delta\delta_\delta \text{ above pole}$$
$$= -\Delta\delta_\delta \text{ below pole} \qquad (6)$$

Plotting the values of $\Delta\delta_\delta$ thus formed, we have the means for drawing a curve for the value of $\Delta\delta_\delta$. Forming hourly, or half-hourly groups from the values of n corrected for $\Delta\delta_\delta$ and combining the mean values of each group we have, as for $\Delta\delta_\delta$,

$$\frac{AE + BW - AW - BE}{4} = +\Delta\delta_\alpha \text{ above pole}$$
$$= -\Delta\delta_\alpha \text{ below pole} \qquad (7)$$

Caution.— For below pole use true α, not $\alpha \doteq 12\text{h}$.

Forming normal equations and solving, we have the values of the coefficients of $\Delta\delta_\alpha$, which, when applied, leave the n's free from the systematic errors of *P.G.C.*, and of refraction.

Now gathering together all these various corrections we have the typical equations

$$\left.\begin{aligned} AE\ n &= +CR + dR_1 + \Delta\delta_\alpha + \Delta\delta_\delta \\ AW\ n &= -CR - dR_1 - \Delta\delta_\alpha - \Delta\delta_\delta \\ BE\ n &= -CR - dR_1 - \Delta\delta_\alpha - \Delta\delta_\delta \\ BW\ n &= +CR + dR_1 + \Delta\delta_\alpha + \Delta\delta_\delta \\ \frac{AE - AW - BE + BW}{4} &= +CR + dR_1 + \Delta\delta_\alpha + \Delta\delta_\delta \end{aligned}\right\} (8)$$

There remains one correction unaccounted for, that for variation of latitude.

Put $\Delta\phi' = \phi_0 - \phi' =$ correction to observed latitude. As this will affect all Z. D.'s on a given night or short stretch* by the same amount, we have

AE and *BW*:

$$\begin{aligned} \text{True } z \text{ above pole} &= z' + \Delta\phi' + K \\ \text{True } z \text{ below pole} &= z'' + \Delta\phi' + K \\ \text{True } z \text{ of pole} &= z_0 + \Delta\phi' + K \\ \text{True Eq. Rdg.} &= z_0 + 90^\circ + \Delta\phi' + K \end{aligned}$$

where K = any constant correction to Z. D. readings.

$$\begin{aligned} \text{Computed True } z &= z_0 + 90^\circ + \Delta\phi - \delta_0 + K \\ \text{Observed True } z &= z' \qquad\quad + \Delta\phi \qquad\ + K \end{aligned}$$

$$\left.\begin{aligned} &AE \text{ and } BW: \\ &(z_0 + 90^\circ - \delta_0) - z' = n = 0.0\,\Delta\phi + 0.0\,K \\ &AW \text{ and } BE: \\ &(z_0 - 90^\circ + \delta_0) - z' = n = 0.0\,\Delta\phi + 0.0\,K \end{aligned}\right\} (9)$$

We see by the above, that our n's are *free from effect of variation of latitude* and any true constant. That is, we have no right to take out the constant that appears in our n's until we have evaluated all our systematic corrections.

*By "stretch" is indicated a continuous series of observations treated as a unit, which may be extended over a period of several days.

Transits

As in the case of zenith distances, so with transits we have several well defined sources of error: the Clock, the Telescope, the Reticule, the Personal Equation, the Atmosphere.

Perhaps the most logical point to start the discussion of reductions of transits is at the stage where the transits have been corrected for chronograph minus eye and ear, magnitude error, pivot error, and collimation and level errors. At this point we must have recourse to the observations of stars and here enter the places of the fundamental catalogue.

If we could obtain azimuth and clock corrections without using places from a fundamental catalogue, we could reduce transits fundamentally. The problem therefore is to obtain values for these corrections that can be shown to be free from fundamental catalogue places. For the zero point in R. A., we may use a fundamental star system, provided we can subsequently free the reductions from the systematic errors of the fundamental catalogue.

Thus the problem resolves itself into one similar to that of the zenith distances, to a study of the effects of the various systematic errors of observation and fundamental catalogue; and, naturally, we will start with the azimuth correction determined from double transits of the same circumpolar star, as we do not

need to know the position of the star used in order to evaluate the azimuth correction.

Put

T' = first transit = Obs'd transit + (Chron − E and E) + Mag. + Pivot + Coll. + Level

T'' = second transit ± 12^h = Obs'd transit ± 12^h + (Chron − E and E) + Mag. + Pivot + Coll. + Level

a' = Azimuth Corr. at first transit.

$A' = \sin z' \sec \delta'$

a'' = Azimuth Corr. at second transit.

$A'' = \sin z'' \sec \delta''$

App' = Apparent Place Correction at first transit.

App'' = Apparent Place Correction at second transit.

$\Delta T'$ = Clock Correction at first transit.

$\Delta T''$ = Clock Correction at second transit.

Put

$$a' = a'' + (a' - a'')$$
$$a'' = a' + (a'' - a')$$

Then

$$T' + A'a' + App' + \Delta T' = \text{True R. A.}$$
$$T'' + A''a'' + App'' + \Delta T'' = \text{True R. A.}$$

To derive a' and a''

$$T' + A'a' + App' + \Delta T' = \text{R. A.}$$
$$T'' + A''[a' + (a'' - a')] + App'' + \Delta T'' = \text{R. A.}$$

$$(T' - T'') + a'(A' - A'') - A''(a'' - a') + (App' - App'') + (\Delta T' - \Delta T'') = 0$$

$$\left.\begin{aligned} a' &= \frac{(T' - T'') + A''(a' - a'') + (App' - App'') + (\Delta T' - \Delta T'')}{A'' - A'} \\ &\text{and, similarly} \\ a'' &= \frac{(T'' - T') + A'(a'' - a') + (App'' - App') + (\Delta T'' - \Delta T')}{A' - A''} \end{aligned}\right\} \quad (10)$$

For $(a' - a'')$ or $(a'' - a')$ use differences from mire readings.

For $(\Delta T' - \Delta T'')$ use −0.5 daily rate from 24^h groups.

For $(\Delta T'' - \Delta T')$ use +0.5 daily rate from 24^h groups.

For $(App' - App'')$ or $(App'' - App')$ use differences between apparent place corrections.

By this method a' and a'' can be found absolutely independent of the system of fundamental stars to be employed. This is a fundamental reduction in every sense of the word. The only assumption is that changes in mires represent changes in azimuth. This does not appear to have been a false assumption for the Albany observations. In the above, mire readings and apparent place corrections are quantities which can be computed independent of the observation. But daily rate depends upon the observations of stars actually made, so it may be well to indicate how the 24^h rates used in the above were obtained.

$$T' + \Delta T' + App' + A'a' = a'_c - dR' + \Delta a'_\alpha + \Delta a'_\delta - (E - W)' - (N - S)'$$
$$T'' + \Delta T'' + App'' + A''a'' = a''_c - dR'' + \Delta a''_\alpha + \Delta a''_\delta - (E - W)'' - (N - S)''.$$

where $(E - W)$ is the difference in transit times depending on whether observed clamp east or clamp west and $(N - S)$ is a similar correction depending on the position of the observer. These are the equations for two successive transits of the same star, 24 hours apart, so in this step we are not confined to primary clock stars.

Now $a'_c = a''_c$, $\Delta a'_\alpha = \Delta a''_\alpha$, $\Delta a'_\delta = \Delta a''_\delta$, $(E - W)' = (E - W)''$, $(N - S)' = (N - S)''$ for the same star, and $dR' = dR''$ within very narrow limits. So we can write

$$(T' - T'') + (App' - App'') + A'(a' - a'') = \Delta T'' - \Delta T' = 24^h \text{ rate} \quad (11)$$

Computing the value of the 24^h rate as above for all stars observed 24 hours apart for the whole stretch, we can form the mean daily rate of the clock absolutely free from all systematic errors of both instrument and fundamental catalogue, and this rate is fundamentally correct, except for the possible error introduced by using differences between mire readings for $(a' - a'')$. We have thus obtained the azimuth correction for each stretch absolutely free from any assumption as to the R. A. of the stars involved, both as to clock rate and circumpolar stars used for azimuth. If these values of a are applied to readings on the mire, we obtain a mean reading of the mire over each stretch and applying this mean reading to the mire readings, we have a set of azimuth corrections to use with all our stars. These azimuth corrections were derived from observa-

tions of the stars and are to be interpolated by means of the mire readings. Correcting the observations of the clock stars for the azimuth of the instrument, we obtain corrected transits of the clock stars from which we are to derive our clock corrections. For this next step, we must have recourse to some fundamental catalogue. The *Preliminary General Catalogue* was used for the Albany observations. We now have *P. G. C.* — corrected transit = ΔT_0 = clock correction. Forming 12^h groups of these ΔT_0's and taking means of the means of two successive 12^h groups, we have a mean clock correction. If, now, we expand this mean clock correction by means of the 24^h rate, derived as explained above, we have a set of clock corrections which are free from $\Delta\alpha_\alpha$ of *P. G. C.*, so far as it follows the "sine cosine" law, and practically free from the effect of dR or any diurnal term.

Returning to the double transits of circumpolar stars we have

$$\begin{aligned} T' + A'a' + \Delta T'_c &= \alpha \\ T'' + A''a'' + \Delta T''_c &= \alpha \end{aligned} \tag{12}$$

where a' and a'' are the values derived from the mire by equations (10).

Collecting the observations of each star and forming the means, we have places for the circumpolar stars derived from our own observations and essentially free from *P. G. C.*, except as to the zero point of R. A. At this stage we can reduce and use observations of all the primary azimuth stars observed in stretches where we were able to determine the azimuth independently from double transits. Using all these observations, we derive Albany positions for 19 primary azimuth stars. We now have the means for determining the azimuth for all stretches where good groups 12 hours apart were observed, as we have Albany places for the azimuth stars, fundamental clock rates, and very nearly fundamental clock corrections. For this purpose all stretches, where good 12^h groups existed, were chosen and the azimuth computed from the 19 primary azimuth stars alone. No south star was used as is so commonly the practice. The formula for computing the azimuths is

$$a' = \frac{\alpha c - Tr' - \Delta Tc'}{A'} \tag{13}$$

where α_c = Albany derived apparent R. A.; Tr' = corrected transit; and ΔT_c = computed clock correction.

Computing the value of a for each observation of the 19 primary azimuth stars, these values of a were plotted and a curve drawn giving values of a for each hour throughout the stretch. These values were applied to all fundamental stars throughout each fundamental stretch, and we may call them the corrected transits. But the effect of dR upon the azimuth corrections was not taken up; in fact, we have no means of evaluating dR until we come to test the meridian by comparison with *P. G. C.* So it is necessary to obtain an expression for the correction of the azimuth curves due to our inability to correct the a's for dR. Here it is well to remember that

$$a = \frac{P.\,G.\,C. - \text{Corrected Transit}}{A}$$

Let us put

a_0 = true azimuth free from dR effect
a' = actual azimuth as computed and affected by dR
$\Delta a'$ = correction to a' on account of dR effect
$a' = a_0 - \Delta a'$

Now

$$a_0 = \frac{\alpha_0 - (Tr. + dR)}{A'} = \frac{\alpha_0 - Tr.}{A'} - \frac{dR}{A'} = a' - \frac{dR}{A'}$$

or

$$-\frac{dR}{A'} = \Delta a'$$

But

$$dR = \sec z' \sec \delta' \, (e\mu + f\mu\rho)$$

hence

$$\Delta a' = \frac{-\sec z' \sec \delta' \, (e\mu + f\mu\rho)}{A'}$$

Let us evaluate the above expression at the pole.

$$\begin{aligned} \Delta a' \text{ at pole} &= \frac{-\sec z' \text{ of pole} \times \sec \delta' \text{ pole } (e\mu + f\mu\rho)}{\sec \delta' \text{ of pole} \times \sin z' \text{ of pole}} \\ &= \frac{-\sec z' \text{ of pole} \times (e\mu + f\mu\rho)}{\sin z' \text{ of pole}} \\ &= -\sec z' \text{ of pole } x \operatorname{cosec} z' \text{ of pole} \times (e\mu + f\mu\rho) \end{aligned}$$

For Albany, z' of pole = 312° 39′.2

hence

$$\sec z' \operatorname{cosec} z' = -2.007$$

or

$$\Delta a' = -\sec z' \operatorname{cosec} z' \, (e\mu + f\mu\rho) = +2.007 \, (e\mu + f\mu\rho)$$

$$\begin{aligned} \text{Corrected Transit} &= \text{Observed Transit} + A'a_0 + dR \\ &= Tr' + A'a' + A'\Delta a' + dR \\ &= Tr' + A'a' + \sec \delta' \sin z' \times 2.007 \, (e\mu + f\mu\rho) \\ &\quad + \sec \delta' \sec z' \, (e\mu + f\mu\rho) \\ &= Tr' + A'a' + \sec \delta' \, (+2.007 \sin z' + \sec z') \times \\ &\quad (e\mu + f\mu\rho) \end{aligned}$$

whence, putting *P. G. C.* R. A. = True or Corrected Transit

$$PGC - (Tr' + A'a') = \sec \delta'(2.007 \sin z' + \sec z') \times (e\mu + f\mu\rho)$$

substituting

$$F' = \sec \delta' \,(2.007 \sin z' + \sec z') \qquad (14)$$

then

$$n = dR = eF'\mu + fF'\mu\rho \qquad (15)$$

which is the formula used for the 22 stretches employed in this preliminary investigation.

Taking into account the second order terms

$$F'' = \sec \delta' \,(+2.0017 \sin z' + \sec z' - .00116 \sec^3 z')$$

and

$$n = dR = eF''\mu + fF''\mu\rho \qquad (16)$$

or, taking into account the conclusions arrived at in Part I where we derived an expression for ρ, the above equation becomes

$$dR = eF''\mu + F''\mu \,[a \sin(\alpha - \odot) + b \cos(\alpha - \odot) + c \sin 2(\alpha - \odot) + d \cos 2(\alpha - \odot)\,] \qquad (17)$$

where $\odot$ = apparent R. A. of the *Sun.*

Let us now proceed to develop the general theory of clock corrections, of systematic errors of instrument and of *P. G. C.*

Let us put

α_c = Apparent predicted R. A. of *P. G. C.*
α_0 = Apparent observed R. A. of Albany observations
ΔT_0 = Observed clock correction
ΔT_c = Computed clock correction
ΔT_T = True clock correction
$\Delta\alpha_\alpha = a \sin \alpha + b \cos \alpha + c \sin 2\alpha + d \cos 2\alpha$
$\Delta\alpha_\delta$ = Correction to *P. G. C.* depending on δ
dR = Differential refraction
$(E - W)$ = Difference in transit times depending on whether observed Clamp East or Clamp West

As stated earlier, I shall assume that the clock runs perfectly until forced, by the observations, to conclude otherwise. As stated in the zenith distance discussion, dR is probably more erratic than all the others, so should be evaluated first. The term $F'\mu\rho$ is nearly equal but of opposite signs 12 hours apart; this is only approximately true, but the fact that it is partly true has helped to minimize its effect in our first approximate clock corrections. The effect of dR upon the 24^h clock rates is practically nil as dR is very nearly the same for the same star observed 24 hours apart.

Now predicted α_c + *P. G. C.* Corrections = Observed α_0 + its corrections.

$$\alpha_c + \Delta\alpha_\alpha + \Delta\alpha_{2\alpha} + \Delta\alpha_\delta = \alpha_0 + dR + (E - W) + \Delta T_T$$
$$\alpha_c - \alpha_0 = \Delta T_0 \doteq dR + (E - W) + \Delta T_T - \Delta\alpha_\alpha - \Delta\alpha_{2\alpha} - \Delta\alpha_\delta$$

For investigation, we will consider a star has been observed every 6 hours, and we will mark them *D* for daylight illumination, and *N* for artificial illumination.

$$\left.\begin{array}{llllll}
1 & \Delta T_0 \;\; 0^h = \Delta T_T + dR + (E - W) - \Delta\alpha_\alpha \;\; 0^h - \Delta\alpha_{2\alpha} \;\; 0^h - \Delta\alpha_\delta & D & & \\
2 & \Delta T_0 \;\; 6^h = \Delta T_T + dR + (E - W) - \Delta\alpha_\alpha \;\; 6^h - \Delta\alpha_{2\alpha} \;\; 6^h - \Delta\alpha_\delta & N & & \\
3 & \Delta T_0 \; 12^h = \Delta T_T + dR + (E - W) - \Delta\alpha_\alpha \; 12^h - \Delta\alpha_{2\alpha} \; 12^h - \Delta\alpha_\delta & N & & \\
4 & \Delta T_0 \; 18^h = \Delta T_T + dR + (E - W) - \Delta\alpha_\alpha \; 18^h - \Delta\alpha_{2\alpha} \; 18^h - \Delta\alpha_\delta & N & & \\
5 & \Delta T_0 \;\; 0^h = \Delta T_T + dR + (E - W) - \Delta\alpha_\alpha \;\; 0^h - \Delta\alpha_{2\alpha} \;\; 0^h - \Delta\alpha_\delta & D & & \\
 & & & & \\
6 & \Delta T_0 \;\; 6^h = \Delta T_T + dR_0 + (E - W) - 0.0 \; \Delta\alpha_\alpha - 1.0 \; \Delta\alpha_{2\alpha} \;\; 0^h - \Delta\alpha_\delta & \text{for } (N \text{ and } D) & \tfrac{1}{2}(1 + 3) & \\
7 & \Delta T_0 \; 12^h = \Delta T_T + dR_0 + (E - W) - 0.0 \; \Delta\alpha_\alpha - 1.0 \; \Delta\alpha_{2\alpha} \;\; 6^h - \Delta\alpha_\delta & \text{for } (N \text{ and } N) & \tfrac{1}{2}(2 + 4) & \\
8 & \Delta T_0 \; 18^h = \Delta T_T + dR_0 + (E - W) - 0.0 \; \Delta\alpha_\alpha - 1.0 \; \Delta\alpha_{2\alpha} \; 12^h - \Delta\alpha_\delta & \text{for } (N \text{ and } D) & \tfrac{1}{2}(3 + 5) & \\
 & & & & \\
9 & \Delta T_0 \;\; 9^h = \Delta T_T + dR_0 + (E - W) - 0.0 \; \Delta\alpha_\alpha - 0.0 \; \Delta\alpha_{2\alpha} \quad - \Delta\alpha_\delta & & \tfrac{1}{2}(6 + 7) & \\
10 & \Delta T_0 \; 15^h = \Delta T_T + dR_0 + (E - W) - 0.0 \; \Delta\alpha_\alpha - 0.0 \; \Delta\alpha_{2\alpha} \quad - \Delta\alpha_\delta & & \tfrac{1}{2}(7 + 8) & \\
 & & & & \\
11 & \Delta T_0 \; 12^h = \Delta T_T + dR_0 + (E - W) - 0.0 \; \Delta\alpha_\alpha - 0.0 \; \Delta\alpha_{2\alpha} \quad - \Delta\alpha_\delta & & \tfrac{1}{2}(9 + 10) & \\
 & & & & \\
12 & \Delta T_0 \; 12^h = \Delta T_T + dR_0 + (E - W) - 0.0 \; \Delta\alpha_\alpha - 1.0 \; \Delta\alpha_{2\alpha} \; 0h - \Delta\alpha_\delta & (N \text{ and } D) & \tfrac{1}{2}(6 + 8) &
\end{array}\right\} A$$

If we assume that these observations 6 hours apart were made on stars of the same declination, then equation (11 A) holds true absolutely, since we have assumed $(E - W)$ to be the $\Delta\alpha_\delta$ of the Albany observations. dR_0 is the mean dR in each step and since dR is of a diurnal nature, the final value of dR_0 should be quite free from $fF'\mu\rho$, and the $eF\mu$ will be of the nature of a constant. So by observing the 6^h groups, we have

$$\Delta T_0\ 12^h = \Delta T_T + \text{a constant} \tag{18}$$

Hence our error in assuming ΔT_0 for $12^h = \Delta T_T$ in a first approximation from 6^h groups is in error by $dR + (E - W) - \Delta\alpha_\delta$ and if we knew these values and could correct for them, we would have ΔT_0 for $12^h = \Delta T_T$. This is a very important point and should be borne in mind.

However, as in the Albany observations no attempt was made to get the 6^h groups, but only the 12^h groups, let us see what can be done. Referring to the second set of equations, seeing we have not the ΔT_0 for 6^h and 18^h in the first set, we have (6A) and (8A) giving (12A), from which we see that the neglect of the intermediate 6h groups has introduced $-1.0\ \Delta\alpha_{2\alpha}\ 0^h$, a term which many astronomers neglect. In the compilation of *P. G. C.* no attempt was made to free each catalogue of this term, only the more glaring cases were investigated and corrected for this term. So the introduction of this term into the value of ΔT_0 12^h above is of minor consequence. Also, in considering ΔT_0 it must be borne in mind that $(E - W)$ and $\Delta\alpha_\delta$ vary with the δ and not with the R. A. So that we have, as terms depending on the time, dR and $\Delta\alpha_{2\alpha}$. Thus we have as an approximate value for ΔT_T, the value $\Delta T_0 = \Delta T_T + dR_0 - \Delta\alpha_{2\alpha}\ 0^h$ and we have for rate, 24^h rate $= 24^h$ rate free from all systematic errors. We have thus, as it were, *introduced a mean clock* whose correction at the mean epoch is wrong by the $dR_0 - \Delta\alpha_{2\alpha}$ of the stars employed but running at a uniform rate, and that rate is not only the true rate but representative of the observations. Hence, if we adopt the value of ΔT_0 as derived from the 12^h groups and expand it by the 24^h rates, we have a straight line to compare with whose only error is a constant. We have a starting point for the investigation of our observations.

Experience has shown that the dR term is by far the most important, so, for each stretch of observing, we have evaluated this term first and corrected the ns for it, where

$$n = P.G.C.\ \alpha - (\text{Obs'd corrd tr.} + \text{Expanded clock corrn}).$$

When n has been corrected for dR, we form $n \cos\delta$ and call this n' and proceed to evaluate $(E - W)$ and $\Delta\alpha_\delta$. We have eliminated the atmospheric error, so we must look to the other three sources of error for an explanation of these two terms. We know *P. G. C.* cannot effect the observations but does introduce a correction $\Delta\alpha_\delta$. We have an expression for the clock behavior. This leaves the telescope as the only source of error not, as yet, investigated. Let us consider $(E - W)$ as the $\Delta\alpha_\delta$ of our observations and caused by the telescope, an assumption which I have found to be very satisfactory and to remove the small difference found between observations clamp E and observations clamp W.

Let us put

$$P.G.C.\ \alpha + \Delta\alpha_\delta = \text{Obsd } \alpha \text{ Cl } E + \Delta\alpha'_\delta \text{ of Obsd } \alpha \text{ Cl } E$$
$$P.G.C.\ \alpha + \Delta\alpha_\delta = \text{Obsd } \alpha \text{ Cl } W + \Delta\alpha''_\delta \text{ of Obsd } \alpha \text{ Cl } W$$

$$P.G.C.\ \alpha - \text{Obsd } \alpha \text{ Cl } E = -\Delta\alpha_\delta + \Delta\alpha'_\delta \text{ Obsd } \alpha \text{ Cl } E = n'_E$$
$$P.G.C.\ \alpha - \text{Obsd } \alpha \text{ Cl } W = -\Delta\alpha_\delta + \Delta\alpha''_\delta \text{ Obsd } \alpha \text{ Cl } W = n'_W$$

From which

$$\frac{n'_E + n'_W}{2} = -\Delta\alpha_\delta + \frac{\Delta\alpha'_\delta \text{ Obsd } \alpha \text{ Cl } E + \Delta\alpha''_\delta \text{ Obsd } \alpha \text{ Cl } W}{2}$$

$$\frac{n'_E - n'_W}{2} = \frac{\Delta\alpha'_\delta \text{ Obsd } \alpha \text{ Cl } E - \Delta\alpha''_\delta \text{ Obsd } \alpha \text{ Cl } W}{2}$$

If the pivots were unequal, it would produce an effect which would be of opposite sign in the two clamps. Any systematic error in the determination of level would change sign with the clamp and a systematic error in azimuth due to instrument would change sign with clamp, if we use south S. D. for clamp E and north Z. D. for clamp W, which amounts to always using the actual zenith distance as read on Circle A.

Then in the above we would have

$\Delta\alpha'_\delta$ Obsd α Cl E S.Z.D. $= -\Delta\alpha''_\delta$ Obsd α Cl W N.Z.D.

and we can use

$$g \sin z + h \cos z = \Delta\alpha'_\delta \text{ Obsd } \alpha \text{ Cl E S. Z. D.}$$
$$-g \sin z - h \cos z = \Delta\alpha''_\delta \text{ Obsd } \alpha \text{ Cl W N. Z. D.}$$

and if, when we combine n'_E and n'_W, we combine according to same star or same declination

$$\frac{n'_E + n'_W}{2} = -\Delta\alpha_\delta + \frac{g \sin z + h \cos z - g \sin z - h \cos z}{2}$$
$$= -\Delta\alpha_\delta \tag{19}$$

$$\frac{n'_E - n'_W}{2} = \frac{g \sin z + h \cos z + g \sin z + h \cos z}{2}$$
$$= g \sin z + h \cos z \qquad (20)$$

This gives perfect elimination as between the $\Delta\alpha_\delta$ of *P. G. C.* and the $\Delta\alpha'_\delta$ of the observations. As will be seen, due to the reversal of direction for stars observed below pole, after $n'_E - n'_W$ has been formed, the signs of all values below pole should be changed before solving for $\frac{1}{2}(E - W) = g \sin z + h \cos z$. This reversal applies to $\frac{1}{2}(E - W)$ because it is instrumental and does not apply to $\Delta\alpha_\delta$. For $\Delta\alpha_\delta$, a curve was drawn. Combining $\frac{1}{2}(E - W)$ and $\Delta\alpha_\delta$ from the curve, n' was corrected for these two; that is, the individual n's and not the group values were corrected giving n''.

The n'''s were then collected in hourly groups and solved for $\Delta\alpha_\alpha$.

$$-\Delta\alpha_\alpha = a \sin \alpha + b \cos \alpha + c \sin 2\alpha + d \cos 2\alpha \qquad (21)$$

By solving the $\Delta\alpha_\alpha$, using n times $\cos \delta$, we can compare the values of this quantity as derived from the zones below pole only, pole to zenith, and zenith to 73° S. Z. D., and from all. Using $n \cos \delta$ in place of n will not materially affect the value determined from the so called clock belt, from which belt this term is generally determined.

By means of the several methods just explained, we are now able to revise our value of ΔT_0 12^h, and obtain a value of clock correction at mean epoch which will differ from absolute clock correction ΔT_T by a vanishing quantity. Doing this, a second approximation should be made as for first approximation. However, in this second approximation we have the means for correcting the n's for all systematic corrections except the particular one under investigation. Working thus in a spiral, our second approximation should give us definitive values for all known systematic corrections. Then, and only then can an examination of the clock itself be made, if evidence has appeared that it is not functioning uniformly. The Albany Riefler No. 218 shows no material deviation from a straight line during the period of a single stretch, so once the systematic errors are evaluated, we have the means for reducing all our fundamental series and forming the Albany places for the *P. G. C.* stars used as fundamentals. Then, with our own places of these stars, we can extend them to the other series not observed fundamentally and produce a catalogue of observed positions of stars, based on Albany fundamental determinations and free from *P. G. C.*, except that we have taken the zero point of *P. G. C.* as our zero point.

The preceding methods so briefly outlined indicate what I consider to be a "Fundamental Reduction" of a series of observations. From a study of the formulæ it will be readily understood that to contribute to the knowledge we already have of the positions of the principal stars, the observations must be so planned and made that we can eliminate certain errors and evaluate others. Observations morning, afternoon, and night must be made whenever the sky will permit; the meteorology must be studied so that we will have observations both north and south of the zenith at the critical times both morning and afternoon to enable us to evaluate dR; double transits of close circumpolars, good 12^h groups, and successive transits of the same stars are absolutely necessary, as well as special observations for determining $(N - S)$, the magnitude correction, and the illumination correction. Above all, it is absolutely essential that the fundamental observations should be carried on for a short period, as a week, by one and the same observer in order that the continuity of the work may not be completely destroyed by the uncertainty of the corrections depending on personal habits. That each observation in fundamental work should be complete, a measure of both R. A. and Decl. is evidently required, for only by having complete observations can the important question of dR be studied.

The methods of reduction outlined are strictly fundamental in declination, but for the right-ascensions no account has been taken of corrections to the equinox. The reduced observations rest upon the system of right-ascensions employed, and are subject to a correction for equinox which may be introduced at any subsequent time from a discussion of *Sun* observations.

It is earnestly to be hoped that in the future fundamental observing programs will be arranged according to the foregoing principles. The observed right-ascensions of the *Sun* depend upon clock corrections derived from the stellar observations. It is therefore highly important that these corrections be determined according to a rigorous method, taking into account all the sources of error involved. Likewise in declination account should be taken of the differential refraction, and an absolute zero point, in determining the declination of the *Sun*. Only through the employment of such methods can we hope to arrive at a correct placing of the equinox.

Until these principles have been incorporated in observing programs it is perhaps as well to base the reduction of current fundamental observations upon some existing fundamental system.

PART III

Example

By Isabella Lange

In order to show clearly how many of the apparent inconsistencies in meridian observations disappear when they are reduced according to the methods outlined in Part II, the detailed reduction of a single stretch of the Albany observations is here presented. The stretch including Series 784–7, June 28 — July 1, 1916 was selected because of the enormous range of the diurnal term in right-ascension, as shown in Plate G and Table I, Part I.

Transits

The observed transits, corrected for chronograph minus eye and ear, fixed minus micrometer wire, magnitude, pivot, collimation and level errors, are given in Column (2), Table L, in which the first line for each star corresponds to the first, and the second line to the second, of two consecutive transits. The daily clock rates are computed from the successive transits of clock stars by (11)*.

$$(T' - T'') + (App' - App'') + A'(a' - a'') = 24^{\text{h}} \text{ rate}$$

T' = corrected transit, first day; T'' = the same, second day; App' = apparent place correction, first day; App'' = the same, second day; A' = azimuth factor. Assuming that the mire represents the changes in azimuth, the mire first day minus the mire second day gives $(a' - a'')$. The mean of the computed daily rates for the individual stars gives the first mean daily rate (Rate_1), for the entire stretch free from any fundamental catalogue.

Using the mire as a measure of the changes in azimuth, and rate_1, the azimuth of the instrument, (az_1, Table M), is obtained from double transits of circumpolar stars by (10).

$$a' = \frac{(T'-T'')+A''(a'-a'')+(App'-App'')+(\Delta T'-\Delta T'')}{A''-A'}$$

Similarly for a''.

When these corrections (az_1) are subtracted from the mire (a_{m}) and the mean is taken we have the mean reading of mire a_{m0}. We then obtain the azimuth of the instrument interpolated by means of the mire (a_1, Table N), by subtracting a_{m0} from the mire curves.

*References are to formulæ in Part II.

TABLE N

	Mire	a_1	a_2	a_3
784				
h	s	s	s	s
1	−.669	−.066	−.047	−.090
2	−.678	−.075	−.048	−.105
3	−.686	−.083	−.048	−.113
785				
13	−.630	−.027	−.070	−.050
14	−.639	−.036	−.081	−.060
15	−.646	−.043	−.095	−.073
16	−.649	−.046	−.105	−.082
17	−.650	−.047	−.107	−.081
18	−.647	−.044	−.102	−.075
19	−.643	−.040	−.097	−.070
1	−.659	−.056	−.064	−.107
2	−.659	−.056	−.059	−.115
3	−.659	−.056	−.055	−.120
786				
9	−.652	−.049	−.038	−.048
10	−.648	−.045	−.036	−.024
11	−.640	−.037	−.035	−.025
13	−.623	−.020	−.034	−.016
14	−.617	−.014	−.034	−.014
15	−.614	−.011	−.036	−.014
16	−.612	−.009	−.040	−.017
17	−.612	−.009	−.046	−.020
18	−.613	−.010	−.053	−.026
19	−.616	−.013	−.059	−.031
20	−.619	−.016	−.062	−.036
1	−.656	−.053	+.004	−.038
2	−.655	−.052	+.015	−.040
3	−.650	−.047	+.022	−.041
4	−.642	−.039	+.026	−.040
787				
13	−.610	−.007	−.003	+.015
14	−.598	+.005	−.010	+.010
15	−.590	+.013	−.018	+.003
16	−.590	+.013	−.030	−.007
17	−.598	+.005	−.041	−.016
18	−.616	−.013	−.051	−.024
19	−.645	−.042	−.059	−.032

a_3 = curve from a_{c3}

TABLE R

784 June 28, 1916 *B. E.* Observer S. A.

Corrd Tr.	$P.G.C._\alpha$	ΔT_0	ΔT_c	n_α	dR_1	dR_2	$n-dR_1$	$n-dR_2$	$n'-dR_2$	$P.G.C._\delta$
h m s	s	s	s	s	s	s	s	s	s	° ′ ″
1 29 23.550	37.019	+13.469	+13.410	+.059	+.002	+.005	+.057	+.054	+.038	+ 88 51 21.39
1 58 32.851	46.164	+13.313	+13.412	−.099	−.018	−.039	−.081	−.060	−.076	+ 41 55 41.70
2 2 14.653	28.018	+13.365	+13.412	−.047	−.024	−.053	−.023	+.006	−.010	+ 23 4 5.92
2 23 29.894	43.247	+13.353	+13.414	−.061	−.030	−.071	−.031	+.010	−.006	+ 8 5 ....
2 34 58.957	72.216	+13.259	+13.414	−.155	−.034	−.083	−.121	−.072	−.088	− 0 2 ....
2 38 45.248	58.479	+13.231	+13.415	−.184	−.032	−.080	−.152	−.104	−.120	+ 2 53 11.33
2 44 50.640	63.924	+13.284	+13.415	−.131	−.022	−.055	−.109	−.076	−.092	+ 26 55 1.71
2 57 41.450	54.816	+13.366	+13.416	−.050	−.032	−.081	−.018	+.031	+.015	+ 3 45 53.01

785 June 29, 1916 *B. E.* Observer S. A.

Corrd Tr.	$P.G.C._\alpha$	ΔT_0	ΔT_c	n_α	dR_1	dR_2	$n-dR_1$	$n-dR_2$	$n'-dR_2$	$P.G.C._\delta$
13 29 24.047	37.595	+13.548	+13.455	+.093	−.002	−.003	+.095	+.096	+.080	+ 91 8 38.59
13 37 1.514	14.925	+13.411	+13.456	−.045	+.022	+.031	−.067	−.076	−.092	− 8 17 4.36
13 43 5.517	19.031	+13.514	+13.456	+.058	+.014	+.020	+.044	+.038	+.022	+ 17 52 20.44
13 44 3.290	16.816	+13.526	+13.456	+.070	+.008	+.012	+.062	+.058	+.042	+ 49 43 54.99
13 48 48.307	61.755	+13.448	+13.456	−.008	+.005	+.008	−.013	−.016	−.032	+ 65 8 18.59
13 50 30.429	43.963	+13.534	+13.457	+.077	+.014	+.020	+.063	+.057	+.041	+ 18 48 56.81
13 57 11.644	25.162	+13.518	+13.457	+.061	+.018	+.026	+.043	+.035	+.019	+ 1 56 47.72
14 1 56.339	69.916	+13.577	+13.457	+.120	+.005	+.008	+.115	+.112	+.096	+ 64 46 39.06
14 11 26.004	39.507	+13.503	+13.458	+.045	+.021	+.031	+.024	+.014	−.002	− 5 36 17.35
14 14 23.348	36.840	+13.492	+13.458	+.034	+.024	+.036	+.010	−.002	−.018	− 12 59 24.64
14 17 41.489	54.909	+13.420	+13.458	−.038	+.077	+.107	−.115	−.145	−.161	− 39 8 9.81
14 27 31.210	44.732	+13.522	+13.459	+.063	+.003	+.004	+.060	+.059	+.043	+ 76 4 12.27
14 29 48.926	62.495	+13.569	+13.459	+.110	−.012	−.018	+.122	+.128	+.112	+107 32 54.43
14 38 27.526	41.104	+13.578	+13.460	+.118	+.021	+.031	+.097	+.087	+.071	− 5 17 52.73
14 41 8.643	22.150	+13.507	+13.460	+.047	+.012	+.019	+.035	+.028	+.012	+ 27 25 33.69
14 49 7.902	21.445	+13.543	+13.460	+.083	+.006	+.010	+.077	+.073	+.057	+ 59 38 5.99
14 50 46.276	59.717	+13.441	+13.460	−.019	+.003	+.005	−.022	−.024	−.040	+ 74 29 57.53
15 7 15.863	29.373	+13.510	+13.461	+.049	+.039	+.045	+.010	+.004	−.012	− 19 28 46.74
15 11 49.026	62.623	+13.597	+13.462	+.135	−.021	−.024	+.156	+.159	+.143	+110 34 31.21
15 14 51.212	64.726	+13.514	+13.462	+.052	+.025	+.029	+.027	+.023	+.007	+ 2 4 44.16
15 30 57.523	71.048	+13.525	+13.463	+.062	+.013	+.020	+.049	+.042	+.026	+ 26 59 42.33
15 33 20.022	33.531	+13.509	+13.463	+.046	+.043	+.066	+.003	−.020	−.036	− 29 30 27.09
15 39 57.619	71.136	+13.517	+13.464	+.053	+.018	+.027	+.035	+.026	+.010	+ 6 41 10.75
15 42 8.309	21.904	+13.595	+13.464	+.131	+.015	+.024	+.116	+.107	+.091	+ 15 40 54.23
15 45 4.054	17.592	+13.538	+13.464	+.074	+.021	+.033	+.053	+.041	+.025	− 3 10 38.29
15 52 24.112	37.659	+13.547	+13.464	+.083	+.015	+.024	+.068	+.059	+.043	+ 15 55 57.64
15 53 36.468	50.017	+13.549	+13.465	+.084	+.038	+.059	+.046	+.025	+.009	− 25 52 40.05
16 9 30.401	43.877	+13.476	+13.466	+.010	−.004	−.006	+.014	+.016	.000	+ 94 40 0.03
16 13 42.617	56.138	+13.521	+13.466	+.055	+.022	+.035	+.033	+.020	+.004	− 4 29 29.84
16 15 55.342	68.892	+13.550	+13.466	+.084	+.037	+.059	+.047	+.025	+.009	− 25 23 46.57
16 18 2.597	16.182	+13.585	+13.466	+.119	+.015	+.024	+.104	+.095	+.079	+ 19 20 52.60
16 19 46.707	60.474	+13.767	+13.466	+.301	+.003	+.005	+.298	+.296	+.280	+ 75 56 58.27
16 22 24.696	38.256	+13.560	+13.466	+.094	+.008	+.012	+.086	+.082	+.066	+ 55 23 43.64
16 24 5.902	19.442	+13.540	+13.467	+.073	+.038	+.062	+.035	+.011	−.005	− 26 15 1.97
16 26 26.318	39.829	+13.511	+13.467	+.044	+.014	+.023	+.030	+.021	+.005	+ 21 40 13.21

TABLE R

784 June 28, 1916 *B. E.* Observer S. A.

Obsd. *Z. D.*	*P.G.C.* *Z.D.*	n_3	$CR+dR_1$	$CR+dR_2$	n'	n''	n'''	μ	ρ	*R*/100
° ′ ″	″	″	″	″	″	″	″			
46 12 8.69	8.46	− .23	+ .32	+ .18	− .55	− .41	− .15	0.983	− .825	+ .59
359 16 24.55	28.77	+4.22	+ .07	+ .01	+4.15	+4.22	+4.48	.979	− .825	− .01
340 24 53.54	52.99	− .55	+ .03	− .04	− .58	− .50	− .24	.978	− .825	− .20
325 26	....	.....	.....	.....	.....	.....	.....	.976	− .825	.....
317 19	....	.....	.....	.....	.....	.....	.....	.974	− .825	.....
320 13 58.63	58.40	− .23	.00	− .08	− .23	− .15	+ .11	.974	− .825	− .47
344 15 48.47	48.78	+ .31	+ .04	− .01	+ .27	+ .32	+ .58	.973	− .825	− .16
321 6 40.11	40.08	− .03	− .00	− .06	− .03	+ .03	+ .29	.971	− .825	− .45

785 June 29, 1916 *B. E.* Observer S. A.

Obsd. *Z. D.*	*P.G.C.* *Z.D.*	n_3	$CR+dR_1$	$CR+dR_2$	n'	n''	n'''	μ	ρ	*R*/100
48 29 25.76	25.66	− .10	+ .17	+ .32	− .27	− .42	− .16	.968	+ .330	+ .63
309 3 41.13	42.71	+1.58	− .20	− .07	+1.78	+1.65	+1.91	.968	+ .330	− .69
335 13 6.86	7.51	+ .65	− .08	− .02	+ .73	+ .67	+ .93	.968	+ .330	− .26
7 4 41.92	42.06	+ .14	+ .02	+ .07	+ .12	+ .07	+ .33	.968	+ .330	+ .07
22 29 5.65	5.66	+ .01	+ .06	+ .12	− .05	− .11	+ .15	.969	+ .330	+ .23
336 9 43.69	43.88	+ .19	− .07	− .03	+ .26	+ .22	+ .48	.969	+ .330	− .25
319 17 35.08	34.79	− .29	− .14	− .08	− .15	− .21	+ .05	.969	+ .330	− .48
22 7 26.31	26.13	− .18	+ .06	+ .10	− .24	− .28	− .02	.969	+ .330	+ .23
311 44 29.62	29.72	+ .10	− .18	− .14	+ .28	+ .24	+ .50	.970	+ .330	− .63
304 21 21.99	22.43	+ .44	− .23	− .18	+ .67	+ .62	+ .88	.970	+ .330	− .81
278 12 36.14	37.26	+1.12	−1.09	− .40	+2.21	+1.52	+1.78	.970	+ .330	−3.63
33 24 59.09	59.34	+ .25	+ .10	+ .12	+ .15	+ .13	+ .39	.971	+ .330	+ .37
64 53 40.59	41.50	+ .91	+ .32	+ .39	+ .59	+ .52	+ .78	.971	+ .330	+1.18
312 2 56.21	54.34	−1.87	− .18	− .18	−1.69	−1.69	−1.43	.971	+ .330	− .62
344 46 20.57	20.76	+ .19	− .04	− .04	+ .23	+ .23	+ .49	.972	+ .330	− .15
16 58 53.38	53.06	− .32	+ .05	+ .05	− .37	− .37	− .11	.972	+ .330	+ .17
31 50 44.45	44.60	+ .15	+ .10	+ .10	+ .05	+ .05	+ .31	.972	+ .330	+ .35
297 52 0.59	0.33	− .26	− .35	− .37	+ .09	+ .11	+ .37	.973	+ .480	−1.05
67 55 17.96	18.28	+ .32	+ .30	+ .30	+ .02	+ .02	+ .28	.973	+ .480	+1.37
319 25 31.29	31.23	− .06	− .16	− .17	+ .10	+ .11	+ .37	.973	+ .480	− .48
344 20 29.62	29.40	− .22	− .05	− .07	− .17	− .15	+ .11	.975	+ .353	− .16
287 50 19.42	19.98	+ .56	− .52	− .73	+1.08	+1.29	+1.55	.975	+ .353	−1.72
324 1 57.31	57.82	+ .51	− .12	− .16	+ .63	+ .67	+ .93	.976	+ .353	− .41
333 1 41.48	41.30	− .18	− .09	− .12	− .09	− .06	+ .20	.976	+ .353	− .29
314 10 8.56	8.78	+ .22	− .17	− .22	+ .39	+ .44	+ .70	.976	+ .353	− .58
333 16 44.78	44.71	− .07	− .08	− .12	+ .01	+ .04	+ .30	.976	+ .353	− .28
291 28 7.15	7.02	− .13	− .42	− .60	+ .29	+ .47	+ .73	.976	+ .353	−1.42
52 0 46.87	47.10	+ .23	+ .20	+ .14	+ .03	+ .09	+ .35	.977	+ .353	+ .72
312 51 17.07	17.23	+ .16	− .18	− .23	+ .34	+ .39	+ .65	.978	+ .353	− .61
291 57 0.23	0.50	+ .27	− .41	− .58	+ .68	+ .85	+1.11	.978	+ .353	−1.38
336 41 39.84	39.67	− .17	− .07	− .10	− .10	− .07	+ .19	.978	+ .353	− .24
33 17 44.87	45.34	+ .47	+ .10	+ .07	+ .37	+ .40	+ .66	.978	+ .353	+ .37
12 44 30.23	30.71	+ .48	+ .03	+ .01	+ .45	+ .47	+ .73	.978	+ .353	+ .13
291 5 45.06	45.10	+ .04	− .43	− .62	+ .47	+ .66	+ .92	.978	+ .353	−1.45
339 0 59.64	60.28	+ .64	− .07	− .09	+ .71	+ .73	+ .99	.978	+ .353	− .22

Corrd Tr.	$P.G.C._\alpha$	ΔT_0	ΔT_c	n_α	dR_1	dR_2	$n-dR_1$	$n-dR_2$	$n'-dR_2$	$P.G.C._\delta$
h m s	s	s	s	s	s	s	s	s	s	° ′ ″
16 27 58.354	71.908	+13.554	+13.467	+.087	+.005	+.008	+.082	+.079	+.063	+ 68 56 59.73
16 30 29.747	43.285	+13.538	+13.467	+.071	+.041	+.067	+.030	+.004	−.012	− 28 2 47.87
16 32 22.174	35.711	+13.537	+13.467	+.070	+.024	+.040	+.046	+.030	+.014	− 10 24 3.20
16 48 5.184	18.775	+13.591	+13.468	+.123	+.016	+.026	+.107	+.097	+.081	+ 15 6 45.83
16 54 23.051	36.582	+13.531	+13.469	+.062	+.001	+.002	+.061	+.060	+.044	+ 82 10 38.28
17 1 18.856	32.454	+13.598	+13.469	+.129	+.016	+.028	+.113	+.101	+.085	+ 12 51 13.69
17 5 24.062	37.590	+13.528	+13.469	+.059	+.028	+.048	+.031	+.011	−.005	− 15 37 27.62
17 8 51.120	64.580	+13.460	+13.470	−.010	+.039	+.068	−.049	−.078	−.094	− 26 53 15.43
17 10 38.951	52.507	+13.556	+13.470	+.086	+.016	+.028	+.070	+.058	+.042	+ 14 29 1.98
17 11 24.690	38.246	+13.556	+13.470	+.086	+.014	+.024	+.072	+.062	+.046	+ 24 56 11.15
17 30 52.061	65.651	+13.590	+13.471	+.119	+.016	+.029	+.103	+.090	+.074	+ 12 37 8.84
17 32 37.089	50.637	+13.548	+13.471	+.077	+.028	+.049	+.049	+.028	+.012	− 15 20 54.62
17 42 7.306	20.824	+13.518	+13.472	+.046	+.041	+.073	+.005	−.027	−.043	− 27 48 6.26
17 59 14.732	28.086	+13.354	+13.473	−.119	.000	−.001	−.119	−.118	−.134	+ 86 36 49.76
18 8 35.128	48.700	+13.572	+13.473	+.099	+.032	+.059	+.067	+.040	+.024	− 21 4 57.90
18 16 48.045	61.607	+13.562	+13.474	+.088	+.021	+.039	+.067	+.049	+.033	− 2 55 20.07
1 29 24.576	38.157	+13.581	+13.501	+.080	+.002	+.005	+.078	+.075	+.059	+ 88 51 21.41
1 48 5.883	19.380	+13.497	+13.502	−.005	−.017	−.046	+.012	+.041	+.025	+ 29 10 19.82
1 49 48.406	61.817	+13.411	+13.502	−.091	−.020	−.052	−.071	−.039	−.055	+ 20 24 2.56
1 58 32.756	46.203	+13.447	+13.503	−.056	−.014	−.038	−.042	−.018	−.034	+ 41 55 41.83
2 2 14.584	28.051	+13.467	+13.503	−.036	−.019	−.053	−.017	+.017	+.001	+ 23 4 . . . .
2 38 45.085	58.507	+13.422	+13.506	−.084	−.005	−.080	−.079	−.004	−.020	+ 2 53 11.54
2 44 50.472	63.956	+13.484	+13.506	−.022	−.003	−.055	−.019	+.033	+.017	+ 26 55 1.82
2 57 41.375	54.843	+13.468	+13.507	−.039	−.005	−.081	−.034	+.042	+.026	+ 3 45 53.21

786 June 30, 1916 *B. E.* Observer S. A.

9 40 53.537	67.004	+13.467	+13.532	−.065	−.002	−.001	−.063	−.064	−.080	+ 24 9 37.76
9 47 47.724	61.218	+13.494	+13.532	−.038	−.002	.000	−.036	−.038	−.054	+ 26 24 7.65
10 3 42.281	55.799	+13.518	+13.533	−.015	−.002	+.003	−.013	−.018	−.034	+ 12 22 33.93
10 11 49.626	63.218	+13.592	+13.534	+.058	−.002	+.004	+.060	+.054	+.038	+ 23 50 6.11
13 29 25.108	38.698	+13.590	+13.547	+.043	−.004	−.003	+.047	+.046	+.030	+ 91 8 38.56
13 43 5.428	19.020	+13.592	+13.548	+.044	+.029	+.020	+.015	+.024	+.008	+ 17 52 20.51
13 48 48.122	61.714	+13.592	+13.548	+.044	+.011	+.007	+.033	+.037	+.021	+ 65 8 18.65
13 57 11.176	24.823	+13.647	+13.549	+.098	+.025	+.018	+.073	+.080	+.064	+ 27 47 24.29
14 1 56.191	69.877	+13.686	+13.549	+.137	+.011	+.008	+.126	+.129	+.113	+ 64 46 39.12
14 8 14.231	27.818	+13.587	+13.549	+.038	+.048	+.033	−.010	+.005	−.011	− 9 53 17.71
14 11 25.936	39.499	+13.563	+13.550	+.013	+.044	+.031	−.031	−.018	−.034	− 5 36 17.31
14 14 23.253	36.832	+13.579	+13.550	+.029	+.051	+.036	−.022	−.007	−.023	− 12 59 24.63
14 28 31.054	44.712	+13.658	+13.550	+.108	+.021	+.015	+.087	+.093	+.077	+ 38 40 27.37
14 30 50.747	64.382	+13.635	+13.551	+.084	+.023	+.018	+.061	+.066	+.050	+ 30 6 28.71
14 35 32.061	45.632	+13.571	+13.551	+.020	+.018	+.014	+.002	+.006	−.010	+ 44 45 57.92
14 36 57.654	71.280	+13.626	+13.551	+.075	+.029	+.022	+.046	+.053	+.037	+ 14 5 7.13
14 38 21.006	34.497	+13.491	+13.551	−.060	+.107	+.082	−.167	−.142	−.158	− 34 49 9.73
14 41 49.597	63.199	+13.602	+13.551	+.051	+.035	+.027	+.016	+.024	+.008	+ 2 14 33.10
14 44 15.544	29.331	+13.787	+13.551	+.236	−.032	−.025	+.268	+.261	+.245	+111 27 31.65
14 52 2.231	15.783	+13.552	+13.552	.000	+.046	+.036	−.046	−.036	−.052	− 11 4 33.65
14 56 18.671	32.242	+13.571	+13.552	+.019	+.043	+.034	−.024	−.015	−.031	− 8 11 26.29
14 58 58.990	72.589	+13.599	+13.552	+.047	+.067	+.052	−.020	−.005	−.021	− 24 57 29.61

Obsd $Z. D.$	$P.G.C.$ $Z.D.$	n_3	$CR+dR_1$	$CR+dR_2$	n'	n''	n'''	μ	ρ	$R/100$
° ′ ″	″	″	″	″	″	″	″			
26 17 47.03	46.80	− .23	+ .07	+ .05	− .30	− .28	− .02	.978	+ .353	+ .28
289 17 59.05	59.20	+ .15	− .48	− .68	+ .63	+ .83	+1.09	.979	+ .353	−1.59
306 56 43.51	43.87	+ .36	− .22	− .29	+ .58	+ .65	+ .91	.979	+ .353	− .75
332 27 33.35	32.90	− .45	− .09	− .11	− .36	− .34	− .08	.980	+ .353	− .29
39 31 27.97	25.35	−2.62	+ .12	+ .10	−2.74	−2.72	−2.46	.980	+ .353	+ .46
330 12 1.16	0.76	− .40	− .10	− .12	− .30	− .28	− .02	.980	+ .353	− .32
301 43 19.68	19.45	− .23	− .27	− .33	+ .04	+ .10	+ .36	.981	+ .353	− .91
290 27 31.33	31.64	+ .31	− .45	− .57	+ .76	+ .88	+1.14	.981	+ .353	−1.50
331 49 49.38	49.05	− .33	− .09	− .11	− .24	− .22	+ .04	.981	+ .353	− .30
342 16 58.17	58.22	+ .05	− .05	− .07	+ .10	+ .12	+ .38	.981	+ .353	− .18
329 57 56.07	55.91	− .16	− .10	− .11	− .06	− .05	+ .21	.982	+ .353	− .33
301 59 51.97	52.45	+ .48	− .27	− .30	+ .75	+ .78	+1.04	.982	+ .353	− .90
289 32 41.05	40.81	− .24	− .48	− .51	+ .24	+ .27	+ .53	.983	+ .353	−1.58
43 57 36.24	36.83	+ .59	+ .15	+ .17	+ .44	+ .42	+ .68	.984	+ .353	+ .55
296 15 49.97	49.17	− .80	− .34	− .31	− .46	− .49	− .23	.984	+ .353	−1.14
314 25 26.14	27.00	+ .86	− .17	− .15	+1.03	+1.01	+1.27	.985	+ .353	− .58
46 12 8.85	8.48	− .37	+ .30	+ .18	− .67	− .55	− .29	.983	− .672	+ .59
346 31 7.47	6.89	− .58	+ .03	− .03	− .61	− .55	− .29	.981	− .672	− .14
337 44 50.40	49.63	− .77	+ .01	− .06	− .78	− .71	− .45	.981	− .672	− .23
359 16 29.35	28.90	− .45	+ .06	+ .01	− .51	− .46	− .20	.980	− .672	− .01
340 25	....	.....	.....	.....	.....	.....	.....	.979	− .672	.....
320 13 57.90	58.61	+ .71	− .08	− .09	+ .79	+ .80	+1.06	.976	− .192	− .47
344 15 48.69	48.89	+ .20	− .01	− .01	+ .21	+ .21	+ .47	.976	− .192	− .16
321 6 41.68	40.28	−1.40	− .07	− .06	−1.33	−1.34	−1.08	.975	− .192	− .45

786 June 30, 1916 *B. E.* Observer S. A.

Obsd $Z. D.$	$P.G.C.$ $Z.D.$	n_3	$CR+dR_1$	$CR+dR_2$	n'	n''	n'''	μ	ρ	$R/100$
341 30 22.88	24.83	+1.95	− .02	+ .30	+1.97	+1.65	+1.91	.952	− .136	− .18
343 44 53.71	54.72	+1.01	− .01	+ .29	+1.02	+ .72	+ .98	.952	− .136	− .16
329 43 22.42	21.00	−1.42	− .05	+ .31	−1.37	−1.73	−1.47	.952	− .136	− .32
341 10 53.32	53.18	− .14	− .02	+ .28	− .12	− .42	− .16	.951	− .136	− .19
48 29 25.77	25.63	− .14	+ .10	+ .33	− .24	− .47	− .21	.960	+ .788	+ .62
335 13 8.71	7.58	−1.13	− .11	− .02	−1.02	−1.11	− .85	.962	+ .788	− .26
22 29 5.35	5.72	+ .37	+ .03	+ .12	+ .34	+ .25	+ .51	.962	+ .788	+ .23
345 8 12.64	11.36	−1.28	− .08	− .00	−1.20	−1.28	−1.02	.964	+ .788	− .15
22 7 25.92	26.19	+ .27	+ .03	+ .11	+ .24	+ .16	+ .42	.964	+ .788	+ .23
307 27 30.36	29.36	−1.00	− .29	− .14	− .71	− .86	− .60	.965	+ .788	− .72
311 44 30.09	29.76	− .33	− .25	− .13	− .08	− .20	+ .06	.966	+ .788	− .62
304 21 22.84	22.44	− .40	− .33	− .17	− .07	− .23	+ .03	.966	+ .788	− .81
356 1 14.98	14.44	− .54	− .04	.00	− .50	− .54	− .28	.968	+ .788	− .04
347 27 15.87	15.78	− .09	− .06	− .02	− .03	− .07	+ .19	.968	+ .725	− .12
2 6 45.84	44.99	− .85	− .02	+ .01	− .83	− .86	− .60	.969	+ .725	+ .02
331 25 54.00	54.20	+ .20	− .12	− .08	+ .32	+ .28	+ .54	.969	+ .725	− .30
282 31 38.15	37.34	− .81	−1.23	− .63	+ .42	− .18	+ .08	.969	+ .725	−2.44
319 35 21.42	20.17	−1.25	− .18	− .13	−1.07	−1.12	− .86	.970	+ .725	− .47
68 48 18.22	18.72	+ .50	+ .20	+ .43	+ .30	+ .07	+ .33	.970	+ .725	+1.43
306 16 14.60	13.42	−1.18	− .29	− .24	− .89	− .94	− .68	.971	+ .725	− .76
309 9 21.83	20.78	−1.05	− .26	− .22	− .79	− .83	− .57	.971	+ .725	− .69
292 23 18.19	17.46	− .73	− .57	− .44	− .16	− .29	− .03	.972	+ .725	−1.35

Corrd Tr.	$P.G.C._\alpha$	ΔT_0	ΔT_c	n_α	dR_1	dR_2	$n-dR_1$	$n-dR_2$	$n'-dR_2$	$P.G.C._\delta$
h m s	s	s	s	s	s	s	s	s	s	° ′ ″
15 7 15.795	29.368	+13.573	+13.552	+.021	+.056	+.045	−.035	−.024	−.040	− 19 28 46.76
15 9 24.436	37.981	+13.545	+13.553	−.008	−.016	−.013	+.008	+.005	−.011	+102 34 21.15
15 12 18.910	32.503	+13.593	+13.553	+.040	+.044	+.035	−.004	+.005	−.011	− 9 4 40.76
15 23 21.007	34.592	+13.585	+13.554	+.031	+.052	+.043	−.021	−.012	−.028	− 16 25 44.57
15 24 11.420	25.066	+13.646	+13.554	+.092	+.023	+.019	+.069	+.073	+.057	+ 29 23 ....
15 29 25.014	38.605	+13.591	+13.554	+.037	+.045	+.037	−.008	.000	−.016	− 9 46 53.42
15 30 57.410	71.040	+13.630	+13.554	+.076	+.024	+.020	+.052	+.056	+.040	+ 26 59 42.49
15 33 19.960	33.527	+13.567	+13.554	+.013	+.081	+.066	−.068	−.053	−.069	− 29 30 27.14
15 39 57.508	71.132	+13.624	+13.555	+.069	+.033	+.027	+.036	+.042	+.026	+ 6 41 10.85
15 44 47.124	60.753	+13.629	+13.555	+.074	+.028	+.023	+.046	+.051	+.035	+ 18 23 53.50
15 46 27.437	41.060	+13.623	+13.555	+.068	+.034	+.029	+.034	+.039	+.023	+ 4 43 37.06
15 52 24.000	37.655	+13.655	+13.555	+.100	+.029	+.024	+.071	+.076	+.060	+ 15 55 57.76
15 53 56.119	69.761	+13.642	+13.556	+.086	+.024	+.020	+.062	+.066	+.050	+ 27 7 8.36
15 55 12.175	25.728	+13.553	+13.556	−.003	+.062	+.052	−.065	−.055	−.071	− 22 23 16.78
16 0 23.258	36.863	+13.605	+13.556	+.049	+.057	+.048	−.008	+.001	−.015	− 19 34 50.00
16 9 30.517	44.109	+13.592	+13.557	+.035	−.007	−.006	+.042	+.041	+.025	+ 94 40 0.21
16 13 42.561	56.138	+13.577	+13.557	+.020	+.040	+.035	−.020	−.015	−.031	− 4 29 29.80
16 26 26.207	39.825	+13.618	+13.558	+.060	+.010	+.023	+.050	+.037	+.021	+ 21 40 13.39
16 53 31.386	45.019	+13.633	+13.560	+.073	+.013	+.029	+.060	+.044	+.028	+ 9 30 10.97
16 57 38.523	52.247	+13.724	+13.560	+.164	+.005	+.012	+.159	+.152	+.136	+ 56 48 40.25
16 59 8.666	22.250	+13.584	+13.560	+.024	+.041	+.094	−.017	−.070	−.086	− 34 0 32.59
17 1 18.792	32.455	+13.663	+13.560	+.103	+.012	+.028	+.091	+.075	+.059	+ 12 51 13.86
17 5 23.986	37.594	+13.608	+13.560	+.048	+.020	+.048	+.028	.000	−.016	− 15 37 27.61
17 8 22.082	35.840	+13.758	+13.561	+.197	+.004	+.009	+.193	+.188	+.172	+ 65 49 4.45
17 10 38.830	52.507	+13.677	+13.561	+.116	+.012	+.027	+.104	+.089	+.073	+ 14 29 2.14
17 11 24.597	38.245	+13.648	+13.561	+.087	+.010	+.024	+.077	+.063	+.047	+ 24 56 11.36
17 16 41.715	55.305	+13.590	+13.561	+.029	+.027	+.064	+.002	−.035	−.051	− 24 55 9.18
17 22 10.787	24.446	+13.659	+13.561	+.098	+.014	+.033	+.084	+.065	+.049	+ 4 12 39.67
17 24 20.186	33.862	+13.676	+13.562	+.114	+.007	+.016	+.107	+.098	+.082	+ 48 19 46.48
17 26 8.058	21.693	+13.635	+13.562	+.073	+.026	+.062	+.047	+.011	−.005	− 23 54 3.47
17 28 21.578	35.319	+13.741	+13.562	+.179	+.006	+.015	+.173	+.164	+.148	+ 52 21 46.00
17 30 51.990	65.654	+13.664	+13.562	+.102	+.012	+.029	+.090	+.073	+.057	+ 12 37 9.00
17 32 37.011	50.643	+13.632	+13.562	+.070	+.020	+.049	+.050	+.021	+.005	− 15 20 54.60
17 43 0.000	13.592	+13.592	+13.563	+.029	+.010	+.023	+.019	+.006	−.010	+ 27 46 5.95
17 59 14.608	27.943	+13.335	+13.564	−.229	.000	−.001	−.229	−.228	−.244	+ 86 36 50.05
18 3 11.955	25.649	+13.694	+13.564	+.130	+.013	+.031	+.117	+.099	+.083	+ 9 33 1.45
18 16 47.982	61.615	+13.633	+13.565	+.068	+.016	+.039	+.052	+.029	+.013	− 2 55 19.95
18 22 38.086	51.696	+13.610	+13.566	+.044	+.028	+.069	+.016	−.025	−.041	− 25 28 10.96
18 25 25.453	39.030	+13.577	+13.566	+.011	+.072	+.097	−.061	−.086	−.102	− 33 2 46.73
18 48 55.384	69.109	+13.725	+13.567	+.158	+.004	+.006	+.154	+.152	+.136	+ 75 20 6.48
19 1 18.918	32.691	+13.773	+13.568	+.205	−.004	−.006	+.209	+.211	+.195	+ 92 48 ....
19 3 55.247	68.422	+13.175	+13.568	−.393	−.002	−.003	−.391	−.390	−.406	+ 89 0 54.51
19 10 14.485	27.931	+13.546	+13.569	−.023	+.051	+.068	−.074	−.091	−.107	− 25 24 5.65
19 13 42.413	56.006	+13.593	+13.569	+.024	+.023	+.031	+.001	−.007	−.023	+ 11 26 36.59
19 13 15.940	29.383	+13.443	+13.569	−.126	−.007	−.010	−.119	−.116	−.132	+ 97 25 21.01
19 21 5.906	19.519	+13.613	+13.569	+.044	+.027	+.035	+.017	+.009	−.007	+ 2 56 49.60
19 25 2.352	15.996	+13.644	+13.569	+.075	+.019	+.025	+.056	+.050	+.034	+ 24 29 39.48
19 27 9.758	23.383	+13.625	+13.569	+.056	+.018	+.023	+.038	+.033	+.017	+ 27 46 57.82

Obsd $Z.\ D.$	$P.G.C.$ $Z.D.$	n_3	$CR+dR_1$	$CR+dR_2$	n'	d''	n'''	μ	ρ	$R/100$
° ′ ″	″	″	″	″	″	″	″			
297 52 1.42	0.31	−1.11	− .42	− .36	− .69	− .75	− .49	.973	+ .725	−1.05
59 55 7.65	8.22	+ .57	+ .16	+ .23	+ .41	+ .34	+ .60	.973	+ .725	+ .96
308 16 7.59	6.31	−1.28	− .27	− .24	−1.01	−1.04	− .78	.973	+ .725	− .71
300 55 4.01	2.50	−1.51	− .37	− .34	−1.14	−1.17	− .91	.975	+ .725	− .93
346 44	....	.....	.....	.....	.....	.....	.....	.975	+ .725	.....
307 33 55.53	53.65	−1.88	− .28	− .27	−1.60	−1.61	−1.35	.975	− .725	− .73
344 20 29.34	29.56	+ .22	− .08	− .07	+ .30	+ .29	+ .55	.976	+ .725	− .16
287 50 20.83	19.93	− .90	− .77	− .72	− .13	− .18	+ .08	.976	+ .725	−1.72
324 1 58.65	57.92	− .73	− .16	− .16	− .57	− .57	− .31	.977	+ .725	− .41
335 44 41.23	40.57	− .66	− .10	− .10	− .56	− .56	− .30	.977	+ .725	− .25
322 4 24.42	24.13	− .29	− .17	− .17	− .12	− .12	+ .14	.978	+ .725	− .44
333 16 45.76	44.83	− .93	− .11	− .11	− .82	− .82	− .56	.978	+ .725	− .28
344 27 55.49	55.43	− .06	− .08	− .07	+ .02	+ .01	+ .27	.978	+ .725	− .16
294 57 31.77	30.29	−1.48	− .49	− .49	− .99	− .99	− .73	.979	+ .725	−1.20
297 45 58.31	57.07	−1.24	− .43	− .43	− .81	− .81	− .55	.979	+ .725	−1.06
52 0 46.79	47.28	+ .49	+ .13	+ .14	+ .36	+ .35	+ .61	.980	+ .725	+ .72
312 51 19.10	17.27	−1.83	− .23	− .24	−1.60	−1.59	−1.33	.981	+ .725	− .61
339 1 0.87	0.46	− .41	− .06	− .09	− .35	− .32	− .06	.982	+ .240	− .22
326 50 57.83	58.04	+ .21	− .10	− .14	+ .31	+ .35	+ .61	.983	+ .240	− .37
14 9 26.36	27.32	+ .96	+ .04	+ .02	+ .92	+ .94	+1.20	.984	+ .240	+ .14
283 20 14.63	14.48	− .15	− .58	−1.03	+ .43	+ .88	+1.14	.984	+ .240	−2.33
330 11 59.65	60.93	+1.28	− .08	− .12	+1.36	+1.40	+1.66	.984	+ .240	− .32
301 43 20.37	19.46	− .91	− .24	− .34	− .67	− .57	− .31	.984	+ .240	− .91
23 9 51.37	51.52	+ .15	+ .07	+ .05	+ .08	+ .10	+ .36	.984	+ .240	+ .24
331 49 49.32	49.21	− .11	− .08	− .11	− .03	.00	+ .26	.984	+ .240	− .30
342 16 58.49	58.43	− .06	− .05	− .07	− .01	+ .01	+ .27	.984	+ .240	− .18
292 25 39.02	37.89	−1.13	− .36	− .51	− .77	− .62	− .36	.984	+ .240	−1.36
321 33 27.14	26.74	− .40	− .12	− .16	− .28	− .24	+ .02	.985	+ .240	− .45
5 40 33.78	33.55	− .23	+ .02	+ .00	− .25	− .24	+ .02	.985	+ .240	+ .06
293 26 45.13	43.60	−1.53	− .34	− .46	−1.19	−1.07	− .81	.985	+ .240	−1.30
9 42 32.90	33.07	+ .17	+ .03	+ .02	+ .14	+ .15	+ .41	.985	+ .240	+ .10
329 57 55.47	56.07	+ .60	− .09	− .11	+ .69	+ .71	+ .97	.985	+ .240	− .33
301 59 52.87	52.47	− .40	− .24	− .30	− .16	− .10	+ .16	.985	+ .240	− .90
345 6 53.42	53.02	− .40	− .04	− .05	− .36	− .35	− .09	.985	+ .240	− .15
43 57 37.08	37.12	+ .04	+ .16	+ .16	− .12	− .12	+ .14	.986	+ .240	+ .55
326 53 48.90	48.52	− .38	− .10	− .11	− .28	− .27	− .01	.986	+ .240	− .37
314 25 27.72	27.12	− .60	− .16	− .16	− .44	− .44	− .18	.987	+ .240	− .58
291 52 37.06	36.11	− .95	− .36	− .34	− .59	− .61	− .35	.987	+ .240	−1.40
284 18 0.89	0.34	− .55	− .83	− .48	+ .28	− .07	+ .19	.987	+ .514	−2.18
32 40 52.13	53.55	+1.42	+ .08	+ .14	+1.34	+1.28	+1.54	.989	+ .514	+ .37
50 9	....	.....	.....	.....	.....	.....	.....	.990	+ .514	.....
46 21 40.39	41.58	+1.19	+ .14	+ .24	+1.05	+ .95	+1.21	.990	+ .514	+ .60
291 56 41.29	41.42	+ .13	− .49	− .21	+ .62	+ .34	+ .60	.991	+ .514	−1.40
328 47 23.94	23.66	− .28	− .12	− .06	− .16	− .22	+ .04	.991	+ .514	− .35
54 46 6.43	8.08	+1.65	+ .19	+ .33	+1.46	+1.32	+1.58	.991	+ .514	+ .81
320 17 37.51	36.67	− .84	− .16	− .08	− .68	− .76	− .50	.992	+ .514	− .47
341 50 26.50	26.55	+ .05	− .07	− .02	+ .12	+ .07	+ .33	.992	+ .514	− .19
345 7 44.82	44.89	+ .07	− .06	− .01	+ .13	+ .08	+ .34	.992	+ .514	− .15

Corrd Tr.	$P.G.C._{\alpha}$	ΔT_0	ΔT_c	n_{α}	dR_1	dR_2	$n-dR_1$	$n-dR_2$	$n'-dR_2$	$P.G.C._{\delta}$
h m s	s	s	s	s	s	s	s	s	s	° ′ ″
1 29 26.228	39.226	+12.998	+13.592	−.594	+.002	+.004	−.596	−.598	−.614	+ 88 51 21.45
1 48 5.856	19.415	+13.559	+13.593	−.034	−.024	−.045	−.010	+.011	−.005	+ 29 10 19.98
1 58 32.702	46.242	+13.540	+13.594	−.054	−.020	−.038	−.034	−.016	−.032	+ 41 55 41.95
2 2 14.538	28.084	+13.546	+13.594	−.048	−.027	−.052	−.021	+.004	−.012	+ 23 4 6.25
2 23 29.836	43.307	+13.471	+13.596	−.125	−.034	−.070	−.091	−.055	−.071	+ 8 5 16.99
2 34 58.733	72.275	+13.542	+13.596	−.054	−.038	−.082	−.016	+.028	+.012	− 0 1 43.37
2 38 45.055	58.537	+13.482	+13.597	−.115	−.036	−.078	−.079	−.037	−.053	+ 2 53 11.75
2 44 50.392	63.989	+13.597	+13.597	.000	−.025	−.055	+.025	+.055	+.039	+ 26 55 1.95
2 52 7.428	20.966	+13.538	+13.597	−.059	−.046	−.100	−.013	+.041	+.025	− 9 13 36.30
2 57 41.277	54.874	+13.597	+13.598	−.001	−.036	−.080	+.035	+.079	+.063	+ 3 45 53.41
3 2 30.115	43.702	+13.587	+13.598	−.011	−.020	−.045	+.009	+.034	+.018	+ 40 38 2.57
3 6 37.561	51.117	+13.556	+13.598	−.042	−.028	−.063	−.014	+.021	+.005	+ 19 24 44.92
787 July 1, 1916 *B. E.* Observer S. A.										
13 29 26.662	39.750	+13.088	+13.638	−.550	−.004	−.003	−.546	−.547	−.563	+ 91 8 38.53
13 37 1.244	14.907	+13.663	+13.639	+.024	+.037	+.031	−.013	−.007	−.023	− 8 17 4.28
13 43 5.298	19.011	+13.713	+13.639	+.074	+.024	+.020	+.050	+.054	+.038	+ 17 52 20.59
13 48 47.917	61.677	+13.760	+13.639	+.121	+.009	+.007	+.112	+.114	+.098	+ 65 8 18.69
13 50 30.210	43.943	+13.733	+13.640	+.093	+.023	+.020	+.070	+.073	+.057	+ 18 48 56.94
13 51 0.201	13.921	+13.720	+13.640	+.080	+.031	+.026	+.049	+.054	+.038	+ 1 27 25.45
13 57 11.110	24.813	+13.703	+13.640	+.063	+.020	+.017	+.043	+.046	+.030	+ 27 47 24.38
14 1 24.542	38.206	+13.664	+13.640	+.024	+.060	+.051	−.036	−.027	−.043	− 26 17 5.21
14 6 5.631	19.341	+13.710	+13.640	+.070	+.005	+.004	+.065	+.066	+.050	+ 74 59 31.99
14 8 14.104	27.811	+13.707	+13.641	+.066	+.038	+.033	+.028	+.033	+.017	− 9 53 17.67
14 11 38.951	52.674	+13.723	+13.641	+.082	+.023	+.020	+.059	+.062	+.046	+ 19 36 59.82
14 13 0.656	14.413	+13.757	+13.641	+.116	+.013	+.011	+.103	+.105	+.089	+ 51 45 15.10
14 14 23.116	36.826	+13.710	+13.641	+.069	+.041	+.035	+.028	+.034	+.018	− 12 59 24.61
14 28 30.963	44.700	+13.737	+13.642	+.095	+.017	+.015	+.078	+.080	+.064	+ 38 40 27.50
14 30 50.679	64.371	+13.692	+13.642	+.050	+.020	+.017	+.030	+.033	+.017	+ 30 6 28.81
14 35 31.927	45.615	+13.688	+13.642	+.046	+.015	+.013	+.031	+.033	+.017	+ 44 45 58.03
14 36 57.534	71.273	+13.739	+13.642	+.097	+.025	+.022	+.072	+.075	+.059	+ 14 5 7.23
14 38 20.846	34.489	+13.643	+13.643	.000	+.091	+.081	−.091	−.081	−.097	− 34 49 9.77
14 41 49.497	63.193	+13.696	+23.643	+.053	+.030	+.027	+.023	+.026	+.010	+ 2 14 33.16
14 49 7.626	21.388	+13.762	+13.643	+.119	+.011	+.009	+.108	+.110	+.094	+ 59 38 6.25
14 52 4.708	18.387	+13.679	+13.643	+.036	+.025	+.022	+.011	+.014	−.002	+ 14 46 57.10
14 56 55.740	69.390	+13.650	+13.644	+.006	+.064	+.058	−.058	−.052	−.068	− 27 44 4.29
15 3 54.536	68.254	+13.718	+13.644	+.074	−.001	−.001	+.075	+.075	+.059	+ 87 33 26.13
15 7 15.664	29.363	+13.699	+13.644	+.055	+.048	+.044	+.007	+.011	−.005	− 19 28 46.76
15 12 18.830	32.499	+13.669	+13.645	+.024	+.038	+.034	−.014	−.010	−.026	− 9 4 40.74
15 17 37.382	51.065	+13.683	+13.645	+.038	+.104	+.096	−.066	−.058	−.074	− 36 33 50.86
15 20 40.701	54.479	+13.778	+13.645	+.133	+.006	+.006	+.127	+.127	+.111	+ 72 7 59.87
15 24 11.332	25.058	+13.726	+13.645	+.081	+.020	+.019	+.061	+.062	+.046	+ 29 23 35.64
15 30 57.333	71.034	+13.701	+13.646	+.055	+.021	+.019	+.034	+.036	+.020	+ 26 59 42.63
15 39 10.698	24.364	+13.666	+13.646	+.020	+.044	+.041	−.024	−.021	−.037	− 15 24 37.15
15 45 3.905	17.586	+13.681	+13.646	+.035	+.060	+.032	−.025	+.003	−.013	− 3 10 38.18
15 46 27.273	41.057	+13.684	+13.646	+.038	+.052	+.028	−.014	+.010	−.006	+ 4 43 37.14
15 49 45.848	59.541	+13.693	+13.646	+.047	+.084	−.051	−.037	+.098	+.082	+119 8 8.76
15 53 36.334	50.013	+13.679	+13.647	+.032	+.107	+.057	−.075	−.025	−.041	− 25 52 40.15

Obsd *Z. D.*	*P.G.C. Z.D.*	n_3	$CR+dR_1$	$CR+dR_2$	n'	n''	n'''	μ	ρ	*R*/100
° ′ ″	″	″	″	″	″	″	″			
46 12 7.95	8.52	+ .57	+ .34	+ .18	+ .23	+ .39	+ .65	.981	− .920	+ .59
346 31 7.99	7.05	− .94	+ .05	− .03	− .99	− .91	− .65	.978	− .920	− .13
359 16 28.68	29.02	+ .34	+ .08	+ .01	+ .26	+ .33	+ .59	.976	− .920	− .01
340 24 53.69	53.32	− .37	+ .04	− .05	− .41	− .32	− .06	.976	− .920	− .20
325 26 5.12	4.06	−1.06	+ .01	− .09	−1.07	− .97	− .71	.973	− .920	− .39
317 19 3.12	3.70	+ .58	+ .01	− .10	+ .57	+ .68	+ .94	.971	− .920	− .51
320 13 59.10	58.82	− .28	+ .01	− .09	− .29	− .19	+ .07	.970	− .920	− .46
344 15 48.59	49.02	+ .43	+ .04	− .01	+ .39	+ .44	+ .70	.969	− .920	− .16
308 7 10.86	10.77	− .09	+ .01	− .11	− .10	+ .02	+ .28	.968	− .920	− .71
321 6 40.78	40.48	− .30	+ .01	− .06	− .31	− .24	+ .02	.967	− .920	− .45
357 58 48.52	49.64	+1.12	+ .08	+ .04	+1.04	+1.08	+1.34	.966	− .920	− .02
336 45 32.09	31.99	− .10	+ .04	− .02	− .14	− .08	+ .18	.966	− .920	− .24
787 July 1, 1916 *B. E.* Observer S. A.										
48 29 25.30	25.60	+ .30	+ .13	+ .34	+ .17	− .04	+ .22	.949	+ .619	+ .62
309 3 43.11	42.79	− .32	− .24	− .05	− .08	− .27	− .01	.950	+ .619	− .67
335 13 8.67	7.66	−1.01	− .10	− .01	− .91	−1.00	− .74	.951	+ .619	− .25
22 29 5.41	5.76	+ .35	+ .04	+ .13	+ .31	+ .22	+ .48	.951	+ .619	+ .23
336 9 43.87	44.01	+ .14	− .09	− .02	+ .23	+ .16	+ .42	.951	+ .619	− .24
318 48 12.42	12.52	+ .10	− .17	− .06	+ .27	+ .16	+ .42	.951	+ .619	− .48
345 8 11.89	11.45	− .44	− .06	.00	− .38	− .44	− .18	.952	+ .619	− .15
291 3 42.50	41.86	− .64	− .55	− .14	− .09	− .50	− .24	.952	+ .619	−1.41
32 20 19.30	19.06	− .24	+ .07	+ .15	− .31	− .39	− .13	.953	+ .619	+ .35
307 27 29.71	29.40	− .31	− .26	− .13	− .05	− .18	+ .08	.953	+ .619	− .71
336 57 47.48	46.89	− .59	− .09	− .04	− .50	− .55	− .29	.953	+ .619	− .23
9 6 2.79	2.17	− .62	+ .00	+ .05	− .62	− .67	− .41	.954	+ .619	+ .09
304 21 23.24	22.46	− .78	− .29	− .16	− .49	− .62	− .36	.954	+ .619	− .80
356 1 16.39	14.57	−1.82	− .03	+ .00	−1.79	−1.82	−1.56	.955	+ .619	− .04
347 27 16.97	15.88	−1.09	− .06	− .02	−1.03	−1.07	− .81	.955	+ .619	− .12
2 6 45.43	45.10	− .33	− .02	+ .02	− .31	− .35	− .09	.956	+ .619	+ .02
331 25 53.87	54.30	+ .43	− .11	− .08	+ .54	+ .51	+ .77	.956	+ .619	− .30
282 31 38.53	37.30	−1.23	−1.08	− .58	− .15	− .65	− .39	.956	+ .619	−2.41
319 35 20.70	20.23	− .47	− .17	− .13	− .30	− .34	− .08	.957	+ .619	− .47
16 58 53.30	53.32	+ .02	+ .02	+ .05	.00	− .03	+ .23	.957	+ .619	+ .17
332 7 44.91	44.17	− .74	− .11	− .09	− .63	− .65	− .39	.958	+ .619	− .29
289 36 43.77	42.78	− .99	− .61	− .48	− .38	− .51	− .25	.958	+ .619	−1.53
44 54 13.29	13.20	− .09	+ .11	+ .15	− .20	− .24	+ .02	.959	+ .619	+ .55
297 52 1.73	0.31	−1.42	− .39	− .35	−1.03	−1.07	− .81	.960	+ .619	−1.04
308 16 7.20	6.33	− .87	− .25	− .24	− .62	− .63	− .37	.960	+ .619	− .70
280 46 56.06	56.21	+ .15	−1.31	−1.16	+1.46	+1.31	+1.57	.960	+ .619	−2.79
29 28 46.87	46.94	+ .07	+ .06	+ .07	+ .01	.00	+ .26	0.961	+ .619	+ .31
346 44 22.65	22.71	+ .06	− .06	− .06	+ .12	+ .12	+ .38	.961	+ .619	− .13
344 20 30.05	29.70	− .35	− .07	− .07	− .28	− .28	− .02	.962	+ .619	− .16
301 56 10.87	9.92	− .95	− .32	− .34	− .63	− .61	− .35	.963	+ .619	− .89
314 10 8.81	8.89	+ .08	− .28	− .22	+ .36	+ .30	+ .56	.963	+1.166	− .57
322 4 23.79	24.21	+ .42	− .21	− .16	+ .63	+ .58	+ .84	.963	+1.166	− .43
76 28 55.49	55.83	+ .34	− .33	+ .28	+ .67	+ .06	+ .32	.964	+1.166	+2.25
291 28 7.39	6.92	− .47	− .81	− .59	+ .34	+ .12	+ .38	.965	+1.166	−1.40

Corrd Tr.	$P.G.C._{\alpha}$	ΔT_0	ΔT_c	n_{α}	dR_1	dR_2	$n-dR_1$	$n-dR_2$	$n'-dR_2$	$P.G.C._{\delta}$
h m s	s	s	s	s	s	s	s	s	s	° ′ ″
15 55 26.617	40.355	+13.738	+13.647	+.091	+.080	+.043	+.011	+.048	+.032	− 16 17 27.14
16 0 23.203	36.862	+13.659	+13.647	+.012	+.087	+.047	−.075	−.035	−.051	− 19 34 50.02
16 9 30.219	44.325	+14.106	+13.648	+.458	−.011	−.006	+.469	+.464	+.448	+ 94 40 0.37
16 6 56.844	70.534	+13.690	+13.647	+.043	+.086	+.047	−.043	−.004	−.020	− 19 14 50.10
16 9 46.401	60.119	+13.718	+13.648	+.070	+.061	+.033	+.009	+.037	+.021	− 3 28 55.05
16 13 42.462	56.138	+13.676	+13.648	+.028	+.062	+.034	−.034	−.006	−.022	− 4 29 29.73
16 26 26.114	39.824	+13.710	+13.649	+.061	+.012	+.023	+.049	+.038	+.022	+ 21 40 13.55
16 36 33.062	46.742	+13.680	+13.649	+.031	+.023	+.047	+.008	−.016	−.032	− 17 35 1.49
16 40 48.011	61.683	+13.672	+13.650	+.022	−.043	−.087	+.065	+.109	+.093	+123 23 24.71
16 54 22.585	36.385	+13.800	+13.651	+.149	+.001	+.002	+.148	+.147	+.131	+ 82 10 38.78
17 5 23.958	37.598	+13.640	+13.651	−.011	+.022	+.047	−.033	−.058	−.074	− 15 37 27.60
17 10 38.794	52.511	+13.717	+13.652	+.065	+.013	+.027	+.052	+.038	+.022	+ 14 29 2.30
17 11 24.552	38.245	+13.693	+13.652	+.041	+.011	+.023	+.030	+.018	+.002	+ 24 56 11.55
17 16 41.616	55.311	+13.695	+13.652	+.043	+.030	+.063	+.013	−.020	−.036	− 24 55 9.23
17 21 4.979	18.642	+13.663	+13.652	+.011	+.030	+.063	−.019	−.052	−.068	− 24 6 5.39
17 22 10.735	24.449	+13.714	+13.652	+.062	+.015	+.032	+.047	+.030	+.014	+ 4 12 39.79
17 28 54.893	68.695	+13.802	+13.653	+.149	−.021	−.045	+.170	+.194	+.178	+115 53 52.44
17 30 51.925	65.660	+13.735	+13.653	+.082	+.013	+.028	+.069	+.054	+.038	+ 12 37 9.16
17 39 9.323	23.047	+13.724	+13.654	+.070	+.015	+.033	+.055	+.037	+.021	+ 4 36 1.19
17 42 59.906	73.594	+13.688	+13.654	+.034	+.011	+.023	+.023	+.011	−.005	+ 27 46 6.17
17 43 30.837	44.545	+13.708	+13.654	+.054	+.016	+.034	+.038	+.020	+.004	+ 2 44 12.69
17 47 14.189	27.997	+13.808	+13.654	+.154	−.030	−.067	+.184	+.221	+.205	+120 7 43.43
17 59 14.319	27.790	+13.471	+13.655	−.184	.000	−.001	−.184	−.183	−.199	+ 86 36 50.
18 3 11.898	25.656	+13.758	+13.655	+.103	+.014	+.031	+.089	+.072	+.056	+ 9 33 1.60
18 16 47.860	61.626	+13.766	+13.656	+.110	+.017	+.038	+.093	+.072	+.056	− 2 55 19.86

TABLE M

	Tr.	A	App.	a_m	Rate_1	dR_2	az_1	az_2				
	h m s	s	s	s	s	s	s	s	s	s	s	s
784	1 29 21.850	−36.16	+8.282	−.673	−.048	−1.853	−.081	−.096	−.592	−.592	−.577	−.577
					+.048		−.042	−.057	−.592		−.577	
785	13 29 26.860	+37.51	+7.706	−.634	−.048	− .739	−.056	−.071	−.578	−.585	−.563	−.570
					+.048		−.081	−.096	−.578		−.563	
785	1 29 22.334	−36.16	+7.144	−.659	−.048	−1.816	−.068	−.083	−.591	−.584	−.576	−.570
					+.048		−.029	−.044	−.591		−.576	
786	13 29 26.383	+37.51	+6.604	−.620	−.048	− .732	+.005	−.009	−.625	−.608	−.611	−.594
							−.031	−.046	−.625		−.610	
786	1 29 25.903	−36.16	+6.076	−.656	−.048	−1.714	−.033	−.055	−.623	−.624	−.601	−.606
787	13 29 26.887	+37.51	+5.552	−.604	+.048	− .724	+.019	+.013	−.623	−.623	−.617	−.617
								a_{mo}		**−.603**		**−.589**

Obsd $Z.D.$	$P.G.C.$ $Z.D.$	n_δ	$CR+dR_1$	$CR+dR_2$	n'	n''	n'''	μ	ρ	$R/100$
° ′ ″	″	″	″	″	″	″	″			
301 3 20.84	19.93	− .91	− .47	− .36	− .44	− .55	− .29	.965	+1.166	− .92
297 45 57.79	57.05	− .74	− .56	− .42	− .18	− .32	− .06	.966	+1.166	−1.05
52 0 47.28	47.44	+ .16	+ .05	+ .14	+ .11	+ .02	+ .28	.968	+1.166	+ .71
298 5 57.30	56.97	− .33	− .55	− .42	+ .22	+ .09	+ .35	.967	+1.166	−1.04
313 51 52.36	52.02	− .34	− .28	− .22	− .06	− .12	+ .14	.968	+1.166	− .58
312 51 18.76	17.34	−1.42	− .29	− .23	−1.13	−1.19	− .93	.969	+1.166	− .60
339 1 0.53	0.62	+ .09	− .06	− .09	+ .15	+ .18	+ .44	.970	+ .273	− .21
299 45 45.32	45.58	+ .26	− .27	− .38	+ .53	+ .64	+ .90	.971	+ .273	− .97
80 44 12.90	11.78	−1.12	+ .99	+ .19	−2.11	−1.31	−1.05	.971	+ .273	+3.26
39 31 25.64	25.85	+ .21	+ .13	+ .10	+ .08	+ .11	+ .37	.972	+ .273	+ .46
301 43 19.58	19.47	− .11	− .25	− .33	+ .14	+ .22	+ .48	.972	+ .273	− .90
331 49 49.71	49.37	− .34	− .08	− .11	− .26	− .23	+ .03	.972	+ .273	− .30
342 16 58.94	58.62	− .32	− .05	− .07	− .27	− .25	+ .01	.972	+ .273	− .18
292 25 36.84	37.84	+1.00	− .37	− .51	+1.37	+1.51	+1.77	.973	+ .273	−1.35
293 14 41.98	41.68	− .30	− .35	− .47	+ .05	+ .17	+ .43	.973	+ .273	−1.29
321 33 27.10	26.86	− .24	− .12	− .15	− .12	− .09	+ .17	.973	+ .273	− .44
73 14 38.36	39.51	+1.15	+ .54	+ .40	+ .61	+ .75	+1.01	.973	+ .273	+1.83
329 57 56.39	56.23	− .16	− .09	− .11	− .07	− .05	+ .21	.973	+ .273	− .32
321 56 48.13	48.26	+ .13	− .12	− .14	+ .25	+ .27	+ .53	.974	+ .273	− .44
345 6 52.71	53.24	+ .53	− .04	− .05	+ .57	+ .58	+ .84	.974	+ .273	− .15
320 4 59.49	59.76	+ .27	− .13	− .15	+ .40	+ .42	+ .68	.974	+ .273	− .47
77 28 32.16	30.50	−1.66	+ .73	+ .61	−2.39	−2.27	−2.01	.974	+ .273	+2.45
43 57	....	.....	.....	.....	.....	.....	.....	.974	+ .273	.....
326 53 48.42	48.67	+ .25	− .10	− .11	+ .35	+ .36	+ .62	.975	+ .273	− .37
314 25 27.13	27.21	+ .08	− .16	− .16	+ .24	+ .24	+ .50	.975	+ .273	− .57

TABLE L

δ	Tr.	App.	A	Mire	a_2	dR_1	dR_2	Rate$_1$	Rate$_2$	Rate$_3$	Rate$_4$
784-785											
°	h m s	s		s	s	s	s	s	s	s	s
+41.9	1 58 32.852	−1.977	+.017	−.678	−.048	−.018	−.039	+.134	+.134	+.130	+.133
	32.757	−2.016		−.659	−.059	−.014	−.038				
+23.1	2 2 14.670	−1.962	+.364	−.678	−.048	−.024	−.053	+.091	+.102	+.097	+.102
	14.605	−1.995		−.659	−.059	−.019	−.053				
+ 2.9	2 38 45.279	−1.714	+.641	−.683	−.048	−.032	−.080	+.170	+.191	+.164	+.191
	45.122	−1.742		−.659	−.056	−.005	−.080				
+26.9	2 44 50.655	−1.818	+.304	−.684	−.048	−.022	−.055	+.190	+.200	+.181	+.200
	50.489	−1.850		−.659	−.056	−.003	−.055				
+ 3.8	2 57 41.480	−1.633	+.629	−.686	−.048	−.032	−.081	+.080	+.102	+.075	+.102
	41.410	−1.660		−.659	−.055	−.005	−.081				
785-786											
+17.9	13 43 5.551	−2.825	+.441	−.636	−.078	+.014	+.020	+.090	+.078	+.073	+.078
	5.443	−2.814		−.619	−.034	+.029	+.020				
− 5.6	14 11 26.067	−3.066	+.750	−.640	−.084	+.021	+.031	+.079	+.060	+.037	+.060
	25.962	−3.058		−.616	−.034	+.044	+.031				
−13.0	14 14 23.419	−3.145	+.847	−.641	−.084	+.024	+.036	+.107	+.086	+.059	+.086
	23.282	−3.136		−.616	−.034	+.051	+.036				

δ	Tr.	App.	A	Mire	a_2	dR_1	dR_2	Rate₁	Rate₂	Rate₃	Rate₄
°	h m s	s	s	s	s	s	s	s	s	s	s
−19.5	15 7 15.953	−3.570	+.938	−.646	−.096	+.039	+.045	+.089	+.063	+.046	+.063
	15.829	−3.565		−.614	−.036	+.056	+.045				
+27.0	15 30 57.553	−3.184	+.303	−.648	−.100	+.013	+.020	+.112	+.104	+.093	+.104
	57.422	−3.176		−.613	−.038	+.024	+.020				
+ 6.7	15 39 57.679	−3.372	+.591	−.648	−.102	+.018	+.027	+.124	+.107	+.092	+.107
	57.530	−3.368		−.613	−.039	+.033	+.027				
+15.9	15 52 24.161	−3.322	+.468	−.649	−.104	+.015	+.024	+.122	+.109	+.095	+.109
	24.018	−3.318		−.612	−.039	+.029	+.024				
− 4.5	16 13 42.684	−3.640	+.735	−.649	−.105	+.021	+.035	+.066	+.046	+.027	+.046
	42.591	−3.640		−.612	−.041	+.040	+.035				
+21.7	16 26 26.359	−3.357	+.385	−.649	−.106	+.014	+.023	+.117	+.107	+.111	+.107
	26.224	−3.353		−.612	−.043	+.010	+.023				
+12.9	17 1 18.911	−3.518	+.510	−.650	−.107	+.016	+.028	+.078	+.066	+.070	+.066
	18.815	−3.519		−.612	−.046	+.012	+.028				
−15.6	17 5 24.156	−4.034	+.883	−.650	−.107	+.028	+.048	+.098	+.079	+.087	+.079
	24.028	−4.038		−.612	−.047	+.020	+.048				
+14.5	17 10 39.003	−3.514	+.487	−.649	−.106	+.016	+.028	+.132	+.121	+.125	+.122
	38.853	−3.514		−.612	−.047	+.012	+.027				
+24.9	17 11 24.726	−3.403	+.336	−.649	−.106	+.014	+.024	+.100	+.093	+.097	+.093
	24.613	−3.402		−.612	−.047	+.010	+.024				
+12.6	17 30 52.114	−3.565	+.513	−.648	−.104	+.016	+.029	+.083	+.073	+.077	+.073
	52.016	−3.568		−.613	−.050	+.012	+.029				
−15.4	17 32 37.180	−4.093	+.879	−.648	−.104	+.028	+.049	+.100	+.083	+.091	+.083
	37.055	−4.099		−.613	−.050	+.020	+.049				
− 2.9	18 16 48.117	−3.860	+.715	−.646	−.101	+.021	+.039	+.081	+.071	+.076	+.071
	48.021	−3.868		−.614	−.055	+.016	+.039				
+29.2	1 48 5.899	−2.058	+.267	−.659	−.060	−.017	−.046	+.080	+.062	+.069	+.061
	5.853	−2.093		−.655	+.013	−.024	−.045				
+41.9	1 58 32.757	−2.016	+.017	−.659	−.059	−.014	−.038	+.094	+.093	+.099	+.093
	32.702	−2.055		−.655	+.015	−.020	−.038				
+23.1	2 2 14.605	−1.995	+.364	−.659	−.059	−.019	−.053	+.104	+.079	+.087	+.078
	14.533	−2.028		−.655	+.015	−.027	−.052				
+ 2.9	2 38 45.122	−1.742	+.641	−.659	−.056	−.005	−.080	+.106	+.062	+.093	+.060
	45.042	−1.772		−.652	+.019	−.036	−.078				
+26.9	2 44 50.489	−1.850	+.304	−.659	−.056	−.003	−.055	+.134	+.113	+.135	+.113
	50.386	−1.883		−.651	+.020	−.025	−.055				
+ 3.8	2 57 41.410	−1.660	+.629	−.659	−.055	−.005	−.081	+.172	+.129	+.160	+.128
	41.263	−1.691		−.650	+.022	−.036	−.080				
786-787											
+17.9	13 43 5.443	−2.814	+.441	−.619	−.034	+.029	+.020	+.125	+.121	+.126	+.121
	5.302	−2.805		−.602	−.008	+.024	+.020				
+27.8	13 57 11.186	−2.857	+.291	−.617	−.034	+.025	+.018	+.058	+.056	+.061	+.057
	11.113	−2.847		−.599	−.010	+.020	+.017				
− 9.9	14 8 14.258	−3.071	+.805	−.617	−.034	+.048	+.033	+.122	+.120	+.130	+.120
	14.113	−3.064		−.597	−.011	+.038	+.033				
−13.0	14 14 23.282	−3.136	+.847	−.616	−.034	+.051	+.036	+.133	+.131	+.141	+.132
	23.126	−3.130		−.596	−.012	+.041	+.035				
+30.1	14 30 50.756	−2.975	+.251	−.615	−.035	+.023	+.018	+.057	+.057	+.060	+.058
	50.683	−2.964		−.594	−.014	+.020	+.017				
+14.1	14 36 57.671	−3.060	+.493	−.615	−.035	+.029	+.022	+.112	+.113	+.117	+.113
	57.541	−3.053		−.593	−.015	+.025	+.022				

δ	Tr.	App.	A	Mire	a_2	dR_1	dR_2	Rate₁	Rate₂	Rate₃	Rate₄
°	h m s	s	s	s	s	s	s	s	s	s	s
+ 2.2	14 41 49.620	−3.161	+.649	−.615	−.035	+.035	+.027	+.093	+.095	+.100	+.095
	49.507	−3.155		−.593	−.016	+.030	+.027				
−19.5	15 7 15.829	−3.565	+.938	−.614	−.036	+.056	+.045	+.120	+.127	+.135	+.128
	15.682	−3.560		−.590	−.019	+.048	+.044				
− 9.1	15 12 18.939	−3.435	+.795	−.614	−.037	+.044	+.035	+.070	+.076	+.082	+.077
	18.846	−3.431		−.590	−.020	+.038	+.034				
+29.4	15 24 11.430	−3.144	+.264	−.613	−.038	+.023	+.019	+.078	+.080	+.083	+.080
	11.338	−3.136		−.590	−.023	+.020	+.019				
+27.0	15 30 57.422	−3.176	+.303	−.613	−.038	+.024	+.020	+.068	+.071	+.051	+.072
	57.340	−3.169		−.590	−.024	+.044	+.019				
+ 4.7	15 46 27.461	−3.414	+.617	−.612	−.039	+.034	+.029	+.054	+.060	+.042	+.061
	27.390	−3.411		−.590	−.027	+.052	+.028				
−19.6	16 0 23.296	−3.872	+.939	−.612	−.040	+.057	+.048	+.043	+.054	+.024	+.055
	23.231	−3.871		−.590	−.030	+.087	+.047				
− 4.5	16 13 42.591	−3.640	+.735	−.612	−.041	+.040	+.035	+.090	+.098	+.076	+.099
	42.486	−3.640		−.592	−.032	+.062	+.034				
+21.7	16 26 26.224	−3.353	+.385	−.612	−.043	+.010	+.023	+.089	+.093	+.091	+.093
	26.127	−3.352		−.593	−.035	+.012	+.023				
−15.6	17 5 24.028	−4.038	+.883	−.612	−.047	+.020	+.048	+.026	+.033	+.031	+.034
	23.995	−4.042		−.599	−.042	+.022	+.047				
+14.5	17 10 38.853	−3.514	+.487	−.612	−.047	+.012	+.027	+.037	+.041	+.040	+.041
	38.815	−3.518		−.602	−.043	+.013	+.027				
+24.9	17 11 24.613	−3.402	+.336	−.612	−.047	+.010	+.024	+.044	+.046	+.045	+.047
	24.566	−3.402		−.602	−.043	+.011	+.023				
−24.9	17 16 41.764	−4.337	+1.020	−.612	−.048	+.027	+.064	+.100	+.105	+.102	+.106
	41.661	−4.343		−.603	−.044	+.030	+.063				
+ 4.2	17 22 10.818	−3.681	+.623	−.612	−.049	+.014	+.033	+.053	+.055	+.054	+.056
	10.763	−3.684		−.604	−.045	+.015	+.032				
+12.6	17 30 52.016	−3.568	+.513	−.613	−.050	+.012	+.029	+.070	+.071	+.070	+.072
	51.949	−3.574		−.607	−.046	+.013	+.028				
+27.8	17 43 0.015	−3.406	+.291	−.613	−.051	+.010	+.023	+.096	+.096	+.095	+.096
	42 59.920	−3.408		−.611	−.048	+.011	+.023				
+ 9.6	18 3 11.984	−3.643	+.554	−.613	−.053	+.013	+.031	+.067	+.064	+.063	+.064
	11.926	−3.650		−.617	−.051	+.014	+.031				
− 2.9	18 16 48.021	−3.868	+.715	−.614	−.055	+.016	+.039	+.141	+.132	+.131	+.133
	47.898	−3.879		−.624	−.053	+.017	+.038				

Sums 51	+4.871	+4.608	+4.496	+4.618
Daily Rate	+ .0955	+ .0904	+ .0882	+ .0905

A selection is now made of the 12ʰ groups for obtaining the computed clock corrections, (ΔT_c, Table O). In forming these groups, stars are so selected that the mean times of the groups are as nearly 12 hours apart as possible, in order to eliminate Δa_α and any diurnal errors. After the azimuth corrections (a_1, Table N) are applied to these stars, the corrected transits are subtracted from Apparent *P. G. C.* to form ΔT_0, Table O. The means of ΔT_0 are now taken for each 12 hour group and the successive means of these are formed. These means are corrected for rate₁ and we have $\Delta T_{c1} = +13^s.523$ for 8ʰ 19ᵐ June 30.

A table of clock corrections is now expanded for every hour using this value of ΔT_{c1} with an hourly rate of $+^s.0040$ (from rate₁). Applying these clock corrections and a_1 to the circumpolar stars, we obtain the first approximation to the positions of the circumpolar stars

TABLE O

	δ	ΔT_{01}	ΔT_{02}	dR_1	dR_2	ΔT_{03}	ΔT_{04}
784							
h	°	s	s	s	s	s	s
2.0	+42	+13.313	+13.313	+.018	+.039	+13.331	+13.352
2.0	+23	+13.375	+13.365	+.024	+.053	+13.389	+13.418
2.4	+ 8	+13.370	+13.353	+.030	+.071	+13.383	+13.424
2.6	0	+13.280	+13.259	+.034	+.083	+13.293	+13.342
2.6	+ 3	+13.251	+13.231	+.032	+.080	+13.263	+13.311
2.8	+27	+13.294	+13.284	+.022	+.055	+13.306	+13.339
3.0	+ 4	+13.387	+13.366	+.032	+.081	+13.398	+13.447
785							
13.8	+19	+13.515	+13.534	−.014	−.020	+13.520	+13.514
14.0	+ 2	+13.489	+13.518	−.018	−.026	+13.500	+13.492
14.2	− 6	+13.470	+13.503	−.021	−.031	+13.482	+13.472
14.2	−13	+13.444	+13.492	−.024	−.036	+13.468	+13.456
14.6	− 5	+13.545	+13.578	−.021	−.031	+13.557	+13.547
14.7	+27	+13.493	+13.507	−.012	−.019	+13.495	+13.488
1.8	+29	+13.496	+13.497	+.017	+.046	+13.514	+13.543
1.8	+20	+13.410	+13.411	+.020	+.052	+13.431	+13.463
2.0	+42	+13.447	+13.447	+.014	+.038	+13.461	+13.485
2.0	+23	+13.466	+13.467	+.019	+.053	+13.486	+13.520
2.6	+ 3	+13.421	+13.422	+.005	+.080	+13.427	+13.502
2.8	+27	+13.484	+13.484	+.003	+.055	+13.487	+13.539
3.0	+ 4	+13.468	+13.468	+.005	+.081	+13.473	+13.549
786							
13.7	+18	+13.584	+13.592	−.029	−.020	+13.563	+13.572
14.0	+28	+13.641	+13.647	−.025	−.018	+13.622	+13.629
14.1	−10	+13.572	+13.587	−.048	−.033	+13.539	+13.554
14.2	− 6	+13.549	+13.563	−.044	−.031	+13.519	+13.532
14.6	+14	+13.615	+13.626	−.029	−.022	+13.597	+13.604
14.7	+ 2	+13.587	+13.602	−.035	−.027	+13.567	+13.575
14.9	− 8	+13.552	+13.571	−.043	−.034	+13.528	+13.537
1.8	+29	+13.577	+13.559	+.024	+.045	+13.583	+13.604
2.0	+42	+13.540	+13.540	+.020	+.038	+13.560	+13.578
2.0	+23	+13.571	+13.546	+.027	+.052	+13.573	+13.598
2.4	+ 8	+13.511	+13.471	+.034	+.070	+13.505	+13.541
2.6	+ 3	+13.526	+13.482	+.036	+.078	+13.518	+13.560
2.8	+27	+13.618	+13.597	+.025	+.055	+13.622	+13.652
3.0	+ 4	+13.640	+13.597	+.036	+.080	+13.633	+13.677
787							
13.8	+19	+13.728	+13.733	−.024	−.020	+13.709	+13.713
14.0	+28	+13.699	+13.703	−.020	−.017	+13.683	+13.686
14.1	−10	+13.693	+13.707	−.038	−.033	+13.669	+13.674
14.2	+20	+13.715	+13.723	−.023	−.020	+13.700	+13.703
14.2	−13	+13.694	+13.710	−.041	−.035	+13.669	+13.675
14.6	+14	+13.727	+13.739	−.025	−.022	+13.714	+13.717
14.7	+ 2	+13.680	+13.696	−.030	−.027	+13.666	+13.669

2 29	+.324			+.310			
8 22	+.408	+.096	+.504		+.416	+.090	+.506
14 16	+.492			+.522			
20 16	+.474	+.048	+.522		+.490	+.045	+.535
2 17	+.456			+.457			
8 18	+.521	.000	+.521		+.528	.000	+.528
14 19	+.586			+.598			
20 20	+.577	−.048	+.529		+.570	−.045	+.525
2 22	+.569			+.542			
8 18	+.637	−.096	+.541		+.629	−.090	+.539
14 15	+.705			+.716			
8 19		ΔT_{c1}	$+13^s.523$			ΔT_{c2}	$+13^s.527$

2 29	+.338			+.376			
8 22	+.421	+.088	+.509		+.436	+.091	+.527
14 16	+.504			+.495			
20 16	+.486	+.044	+.530		+.504	+.045	+.549
2 17	+.468			+.514			
8 18	+.515	.000	+.515		+.543	.000	+.543
14 19	+.562			+.572			
20 20	+.566	−.044	+.522		+.586	−.045	+.541
2 22	+.571			+.601			
8 18	+.629	−.088	+.541		+.646	−.091	+.555
14 15	+.687			+.691			
8 19		ΔT_{c3}	$+13.523$			ΔT_{c4}	$+13^s.543$

TABLE P

$+88^\circ.9$	1^h	29^m	$45^s.230$
+85 .3	4	9	45 .192
+87 .2	7	1	35 .424
+82 .6	7	13	51 .576
+87 .6	15	4	0 .193
+82 .2	16	54	31 .819
+86 .6	17	59	20 .681
+89 .0	19	3	51 .576

Table P contains the R. A.'s of azimuth stars derived at this stage of the work from the observations on all the series when double transits were obtained. These positions are used in computing the azimuth for each star (ac_1, Table Q), formula (13)

$$a' = \frac{\alpha c - (T' + \Delta T'c)}{A'}$$

The values of a_{c1} are now plotted and curves drawn to interpolate the spaces between the azimuth stars, giving a second approximation of azimuth (a_2, Table N). Substituting azimuth from curves (a_2) for azimuth through mire (a_1) in Tables L and O, we have the second approximation to clock rate, $+^s.0904$ and to ΔT_c, $+13^s.527$.

TABLE Q

	a_{c1}	az_1	a_{c2}	az_2
1^h 29^m	−.047	−.081	−.098	−.096
13 29	−.074	−.049	−.054	−.064
16 9	−.104	.....	−.080	.....
16 54	−.121	.....	−.097	.....
17 59	−.092	.....	−.065	.....
1 29	−.062	−.074	−.112	−.090
13 29	−.035	−.012	−.016	−.026
16 9	−.038	.....	−.014	.....
17 59	−.033	.....	−.006	.....
19 1	−.038	.....	−.011	.....
19 3	−.051	.....	−.024	.....
19 13	−.080	.....	−.053	.....
1 29	+.009	−.032	−.038	−.050
13 29	−.023	+.019	−.004	+.013
15 3	−.024	.....	−.003	.....
16 9	+.014	.....	+.037	.....
16 54	−.070	.....	−.045	.....
17 59	−.034	.....	−.007	.....

The entire second approximation including azimuth from curves, and rate$_2$ and ΔT_{c_2} depending on azimuth from curves, may be omitted in our reductions. The mire readings, we have found by a recent study, are a far better measure of the changes in azimuth than was at first supposed. It may be well to mention here, that the mire is placed fairly close to the transit house, about 300 feet, thus eliminating any refraction error in the mire readings that would arise from a mire placed a few miles away as suggested by some observers. The azimuths from curve (a_2) are applied to the corrected transits of all the fundamental stars; the corrected transits are subtracted from apparent *P. G. C.* to form ΔT_0, Table R; and from ΔT_0 the expanded clock corrections ΔT_{c_2} are subtracted to form $n\alpha$.

It will be understood that it would be utterly impossible to evaluate such systematic corrections as $\Delta\alpha_\alpha$, $\Delta\alpha_\delta$ and East minus West, using only one series, so we examine for dR first, as the effect of differential refraction is the most erratic and, at the same time, the largest of our errors. The formula for the factor of this term in right-ascension, taking into account the second order term, is $F' = \sec\delta \sec z(1 - .00116 \sec^2 z)$. Owing to our inability to correct the transits for dR before computing the azimuth correction from formula (13), we must take into account the error due to the dR introduced into the azimuth corrections. The formula, corrected for this error, $F'' = \sec \delta' \,(+2.0017 \sin z' + \sec z' - .00116 \sec^3 z)$ was used in computing dR for this series. Table S gives values of F'' for transits and F' for Z. D.'s in which second order terms have been included.

In Table T, column (1) contains the log correction of log (μ tan z) for γ and $\beta + T$ in the Pulkova Refraction Tables III, V, VII. Column (2) is the sum of the log corrections of γ and $\beta + T$. The relative refraction, μ, is the natural number corresponding to the logarithm in column (2). The hourly rate of change of the refraction, ρ, is the hourly rate of change of μ. For convenience in the solution ρ is multiplied by 100.

TABLE S

N.Z.D.	F''_α	F'_δ	N.Z.D.	F''_α	F'_δ
82	−8.40	+42.46	360	+ 1.36	+ 1.00
80	−6.66	+29.41	350	+ 1.62	+ 1.03
78	−5.34	+21.32	340	+ 1.89	+ 1.13
76	−44.1	+16.10	330	+ 2.20	+ 1.33
74	−3.68	+12.59	320	+ 2.59	+ 1.70
72	−3.10	+10.11	310	+ 3.11	+ 2.41
70	−2.63	+ 8.31	300	+ 3.90	+ 3.95
65	−1.77	+ 5.50	295	+ 4.50	+ 5.50
60	−1.18	+ 3.95	290	+ 5.38	+ 8.31
55	− .73	+ 3.02	288	+ 5.86	+10.11
50	− .39	+ 2.41	286	+ 6.44	+12.59
Pole 47 21			284	+ 7.18	+16.10
45	− .11	+ 1.99	282	+ 8.13	+21.32
40	+ .13	+ 1.70	280	+ 9.45	+29.41
30	+ .51	+ 1.33	278	+11.30	+42.46
20	+ .82	+ 1.13			
10	+1.10	+ 1.03			
0	+1.36	+ 1.00			

TABLE T

	(1)	(2)	h m	μ	ρ
784					
γ	−.01164	−.00805	1 40	.9816	
$\beta+\tau$	+ 359				−.825
	−.01644	−.01296	3 0	.9706	
	+ 348				
785					
	−.01659	−.01417	13 35	.9679	
	+ 242				+.330
	−.01479	−.01221	14 55	.9723	
	+ 258				+.480
	−.01359	−.01094	15 30	.9751	
	+ 265				+.353
	−.00984	−.00665	18 15	.9848	
	+ 319				
	−.01239	−.00808	1 40	.9816	
	+ 431				−.672
	−.01479	−.01056	2 30	.9760	
	+ 423				−.192
	−.01509	−.01090	2 55	.9752	
	+ 419				
786					
	−.02406	−.02134	9 45	.9520	
	+ 272				−.136
	−.02451	−.02175	10 20	.9512	
	+ 276				
	−.02064	−.01815	13 20	.9591	
	+ 249				+.788
	−.01659	−.01399	14 30	.9683	
	+ 260				+.725
	−.01059	−.00782	16 25	.9822	
	+ 277				+.240
	−.00863	−.00570	18 25	.9870	
	+ 293				+.514
	−.00608	−.00305	19 35	.9930	
	+ 303				

−.01209	−.00883	1 35	.9799	
+ 326				−.920
−.01494	−.01187	2 20	.9730	
+ 307				−.920
−.01885	−.01603	3 20	.9638	
+ 282				
787				
−.02302	−.02216	13 40	.9503	
+ 86				+.619
−.01734	−.01628	15 45	.9632	
+ 106				+1.166
−.01434	−.01323	16 20	.9700	
+ 111				+.273
−.01179	−.01068	18 25	.9757	
+ 111				

A least square solution is now made for $dR = eF''\mu + fF''\mu\rho = n$ which gives

$$+^{s}.0014\ F''\mu + {}^{s}.0174\ F''\mu\rho = n.$$

(In the formation of normal equations the weights are reduced for stars of low Z. D.'s using table of weights, App. III, page 281 *P. G. C.*) Expanding these values we get dR_1 (Table R).

It will be noticed by comparing dR_1 with n_α that there is an excellent agreement of signs throughout the entire series, but it will also be seen that the corrections for daytime stars, while they agree in signs, are not large enough. As was pointed out in Part I, this is largely due to using the readings of the shade thermometer in place of the *Sun* thermometer. As we have no means of getting the *Sun* temperatures, we use the formula that represents them. A least square solution of the form

$$eF''\mu + F''\mu\,(a \sin T + b \cos T + c \sin 2T + d \cos 2T) = n$$

where $T = \alpha - \odot$ gives

$$F''\mu\,(-^{s}.0044 +^{s}.0135 \sin T -^{s}.0186 \cos T +^{s}.0068 \sin 2T -^{s}.0002 \cos 2T) = dR_2$$

Expanding for these values we have dR_2, Table R. Comparing dR_2 with dR_1 it is seen that the values for the daytime stars have been increased while the values for the night stars are practically the same.

TABLE U

α	Obs'ns	A.M.T.	dR_1	dR_2	n_α	$n - dR_1$	$n - dR_2$	$n' - dR_2$
h m		h m	s	s	s	s	s	s
2 28	7	20 0	−.027	−.066	−.104	−.076	−.038	−.054
13 47	4	7 15	+.017	+.024	+.038	+.021	+.014	−.002
14 24	5	7 52	+.031	+.045	+.041	+.010	−.004	−.020
15 35	9	9 3	+.025	+.036	+.070	+.045	+.034	+.018
16 26	8	9 54	+.026	+.042	+.080	+.054	+.038	+.022
17 28	10	10 56	+.025	+.045	+.078	+.053	+.033	+.017
2 17	7	19 45	−.012	−.058	−.048	−.036	+.010	−.006
9 55	4	3 19	−.002	+.001	−.015	−.013	−.016	−.032
13 50	2	7 14	+.027	+.019	+.071	+.044	+.052	+.036
14 34	12	7 58	+.044	+.033	+.035	−.009	+.002	−.014
15 34	13	8 58	+.041	+.035	+.055	+.013	+.010	+.004
16 30	5	9 54	+.032	+.046	+.045	+.013	−.001	−.017
17 20	10	10 44	+.016	+.038	+.075	+.059	+.037	+.021
18 16	4	11 40	+.032	+.059	+.063	+.031	+.004	−.012
19 19	5	12 43	+.028	+.036	+.035	+.007	−.001	−.017
2 33	11	19 57	−.030	−.064	−.049	−.019	+.015	−.001
13 47	5	7 7	+.027	+.023	+.067	+.040	+.044	+.028
14 29	11	7 49	+.039	+.035	+.053	+.011	+.018	+.002
15 33	10	8 53	+.057	+.041	+.047	−.010	+.006	−.010
16 15	6	9 35	+.055	+.039	+.041	−.014	+.002	−.014
17 26	12	10 46	+.017	+.037	+.055	+.038	+.018	+.002

TABLE U_1

4	3 19	−.002	+.001	−.015	−.013	−.016	−.032
5	7 7	+.027	+.023	+.067	+.040	+.044	+.028
2	7 14	+.027	+.019	+.071	+.044	+.052	+.036
4	7 15	+.017	+.024	+.038	+.021	+.014	−.002
11	7 49	+.039	+.035	+.053	+.011	+.018	+.002
5	7 52	+.031	+.045	+.041	+.010	−.004	−.020
12	7 58	+.044	+.033	+.035	−.009	+.002	−.014
10	8 53	+.057	+.041	+.047	−.010	+.006	−.010
13	8 58	+.041	+.035	+.055	+.013	+.010	+.004
9	9 3	+.025	+.036	+.070	+.045	+.034	+.018
6	9 35	+.055	+.036	+.041	−.014	+.034	+.018
5	9 54	+.026	+.042	+.045	+.054	+.038	+.022
8	9 54	+.032	+.046	+.080	+.013	−.001	−.017
10	10 44	+.016	+.038	+.075	+.059	+.037	+.021
12	10 46	+.017	+.037	+.055	+.038	+.018	+.002
10	10 56	+.025	+.045	+.078	+.053	+.033	+.017
4	11 40	+.032	+.059	+.063	+.031	+.004	−.012
5	12 43	+.028	+.036	+.035	+.007	−.001	−.017
7	19 45	−.012	−.058	−.048	−.036	+.010	−.006
11	19 57	−.030	−.064	−.049	−.019	+.015	−.001
7	20 0	−.027	−.066	−.104	−.076	−.038	−.054

In Table U are exhibited the hourly means for dR_1, dR_2, n_a, $n - dR_1$, $n - dR_2$ and $n' - dR_2$. In $n' - dR_2$ we use ΔT_c corrected for dR (ΔT_{c4}, Table O). In this table for showing the diurnal effect, the north stars are omitted as there are no day observations north except for circumpolar stars. In U_1 are shown the same means arranged according to mean time. These tables require very little explanation. The reader will see immediately that where we had a solid column of plus n's for night, and minus n's for day, the corrected values are now evenly divided between plus and minus. It will probably be interesting at this point to show how the probable errors have been reduced when the stars have been corrected for dR. In the first line below, the n's have not been multiplied by cosine, but the stars within 10° of the pole have been omitted on account of the large secants. In the second line the n's have been multiplied by the cosine.

	n_a	$n - dR_1$	$n - dR_2$
$.8453 \times \frac{[v]}{m}$	$= \pm^s.058$	$= \pm^s.049$	$= \pm^s.042$
p. e. $\cos\delta$	$= \pm^s.049$	$= \pm^s.035$	$= \pm^s.030$

Now that the effect of dR on the stars as a whole has been shown, let us examine the effect of this correction on the 24^h rates, 12^h groups, and azimuth corrections. The 24^h rates corrected for dR_1 and dR_2 (Table L) have not been materially changed as we computed our rates in such a form as to eliminate this error.

In Table O, dR_1 and dR_2 are applied to ΔT_{02} to form ΔT_{03} and ΔT_{04}; then ΔT_{c3} and ΔT_{c4} are computed. Although we have applied corrections ranging from $+^s.036$ to $-^s.038$ for dR_1 and from $+^s.081$ to $-^s.036$ for dR_2 the mean value for ΔT_{c3} has been changed by only $-^s.004$ and that for ΔT_{c4} by $^s+.016$, showing that the method of eliminating this term in the first approximation was very good. The greatest improvement here is not in the final computed values but in harmoninizing the 12^h groups.

In the following table are given the means of each 12^h groups for ΔT_{02}, ΔT_{03}, and ΔT_{04} in columns 1, 2, and 3. Correcting these for rate we have, 4, 5, and 6 and taking out the means we have 7, 8, and 9.

Column 7 shows the diurnal term in our original ΔT_0's; column 8 shows the diurnal term greatly reduced by dR_1 and in column 9, corrected for dR_2 a still greater portion of it has been taken out.

A study of the azimuth correction will prove interesting. In Table Q, a_{c1} and a_{c2} have been computed from azimuth stars using Albany positions and the formula $a = \dfrac{a_0 - T - \Delta T_c}{A}$. az_1 and az_2 have been computed from double transits using mire curves. a_{c2} and az_2 have been corrected for dR_2. Compare a_{c1} and az_1. Notice that the changes in azimuth from

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
$+^s.310$	$+^s.338$	$+^s.376$	$+^s.422$	$+^s.448$	$+^s.490$	$-^s.107$	$-^s.078$	$-^s.051$
$+.522$	$+.504$	$+.495$	$+.589$	$+.570$	$+.563$	$+.060$	$+.044$	$+.022$
$+.457$	$+.468$	$+.514$	$+.479$	$+.490$	$+.537$	$-.050$	$-.036$	$-.004$
$+.598$	$+.562$	$+.572$	$+.576$	$+.540$	$+.549$	$+.047$	$+.014$	$+.008$
$+.542$	$+.571$	$+.601$	$+.475$	$+.505$	$+.533$	$-.054$	$-.021$	$-.008$
$+.716$	$+.687$	$+.691$	$+.604$	$+.577$	$+.577$	$+.075$	$+.051$	$+.036$
	Means		$+^s.529$	$+^s.526$	$+^s.541$			

one transit of Polaris to the next for a_{c1} do not agree with changes in azimuth for az_1 through mire. Now compare az_1 with az_2 and a_{c2} corrected for dR and notice how well the changes in azimuth computed with Albany R. A.'s and those computed through the mire agree. The following table will help to bring out this point.

	a_{c1}	az_1	a_{c2}	az_2
1 29				
	$-.027$	$+.032$	$+.044$	$+.032$
13 29				
	$-.012$	$-.025$	$-.058$	$-.026$
1 29				
	$-.027$	$+.062$	$+.096$	$+.064$
13 29				
	$-.044$	$-.020$	$-.022$	$-.024$
1 29				
	$+.032$	$+.051$	$+.034$	$+.063$
13 29				

This alternation of signs for observations 12 hours apart has been long known here and we hope, when all the fundamental stretches have been reduced, to be able to offer a solution. Until then, it would be useless to even suggest an explanation.

Zenith Distances

Before a study of systematic corections is made, all the zenith distances of the fundamental stars are corrected for division-error, error of runs, sine-flexure, curvature, inclination of zenith-distance wire, vertical refraction, nadir reading and north minus south.

The double transits of the circumpolar stars are corrected for apparent place and the means of the successive pairs are taken. The mean of these values gives the zenith distance at pole 47° 20′ 47″.07 (Table V), from which the equator point 317° 20′ 47″.07 is derived.

In Table R, the apparent *P. G. C.* zenith distances are formed by adding the equator point to the apparent declination of *P. G. C.* Then the observed Z. D.'s are subtracted from *P. G. C.* Z. D.'s to form $n\delta$.

TABLE V

			$CR + dR$	
	h m	° ′ ″	″	″
784	1 29	46 12 12.17	+.18	12.35
		47 20 47.235		47.485
785	13 29	48 29 22.30	+.32	22.62
		47 20 47.305		47.555
785	1 29	46 12 12.31	+.18	12.49
		47 20 47.325		47.580
786	13 29	48 29 22.34	+.33	22.67
		47 20 46.855		47.110
786	1 29	46 12 11.37	+.18	11.55
		47 20 46.635		46.905
787	13 29	48 29 21.90	+.34	22.26
Zenith distance at pole		47 20 47.07		47.33
Equator point		317 20 47.07		47.33

Solutions of the same form as in R. A. are made and the equations for the stars of low Z. D.'s are given reduced weight, (see *P. G. C.*). The formula for F' is

$$\sec^2 z\,(1.00232 - .003486 \sec^2 z) = F'$$

The values from these solutions are

$$CR + dR_1 = +''.282R + ''.021\,F'\mu - ''.069\,F'\mu\rho$$

using values for ρ derived from the meterorogy as actually recorded, and

$$CR + dR_2 = +''.295R + ''.110F'\mu + F'\mu(+''.067 \sin T + ''.137 \cos T + ''.083 \sin 2T + ''.043 \cos 2T)$$

where the expression for ρ as derived in Part I is used.

The expanded values for $CR + dR_1$ and $CR + dR_2$ are found in Table R; also,

$$n - (CR + dR_1) = n' \qquad n - (CR + dR_2) = n''$$

The dR is now applied to the double transits (Table V) and a new equator-point, 317° 20′ 47″.33, computed.

This equator-point is used in forming new n's for a second approximation to dR.

Since practically all of the dR term is in right ascension for this stretch, the zenith distances do not offer as striking a showing for individual stars. However the equator-points derived from the north and south, corrected for dR_2, are brought together and are made to agree with the equator-point from the double transits.

In the following table column 1 gives the number of observations, column 2 the equator-points from the original observations and column 3 the equator-points corrected for $CR + dR_2$.

	(1)	(2)	(3)
Double Tr.	5	317° 20′ 47″.07	47.33
North	47	47 .18	47.28
South	155	46 .81	47.25

After each stretch has been corrected for dR, as shown in this example, the next step would be to evaluate Δa_α, Δa_δ, $E - W$, $\Delta\delta_\alpha$, and $\Delta\delta_\delta$ using the stars from all the fundamental stretches. Modifying the 12^h groups, ΔT_{C4}, and n_α, for Δa_α, Δa_δ, and $E - W$, and n_δ for $\Delta\delta_\alpha$ and $\Delta\delta_\delta$, the second approximation for dR should be made. When the transits and Z. D.'s are corrected for this final dR, an investigation can be made for personal errors, irregularities in the clock rate, and for variation of the latitude. As we have used one stretch only the methods to be employed for investigating the above enumerated systematic corrections are to be inferred from Part II. But this example shows that our theory of dR as developed in Part I is fully substantiated by the observations and that it is real and very important. Also, the great improvement in the various constants of reduction when the expression for ρ is used over that when ρ from the Albany meteorology is used, cannot fail to attract the attention of the reader.

This article is the result of many years of labor spent in the attempt to disentangle the very perplexing phenomena that develop when daylight observations are compared with night observations. Except for the example, the concluded results in Part I are based upon solutions using the value of ρ derived from Albany shade temperatures. It was only when we came to gather all the material together for final discussion that we felt warranted in using the single and double sine and cosine term for ρ. That this conclusion, which is the keynote to the present discussion, was fully warranted is sufficiently brought out in the example.

In closing, I wish to express my appreciation of the interest taken in the investigation and the many facilities afforded me by the Director. To Dr. R. W. Willson of Harvard College I feel especially indebted for the encouragement he gave me in the early days of this investigation as well as for several very helpful suggestions he made after examining Parts I and II. Also I wish to express my thanks to the various members of the staff who assisted; to Prof. H. Raymond who so carefully read and checked Part II; to Dr. R. E. Wilson who so ably assisted in the preparation and editing of the article; and to Miss Isabella Lange, who has collaborated with me in the investigation from its very inception. The preparation and discussion of the example in Part III is entirely her work. Miss Marie Lange, besides regular computing, has solved all the normal equations. Miss Benjamin, Miss Kampf, Miss Willstaedt, Miss Cramer, Miss Lasch and Miss Vosburgh have taken part in the routine computations of this preliminary investigation.

DEPARTMENT OF MERIDIAN ASTROMETRY
CARNEGIE INSTITUTION OF WASHINGTON
Dudley Observatory, Albany, N. Y.
1922.

THE NICHOLS PRESS, 112 MARKET ST., LYNN

[redacted]